1. Show that \( f(z) = 4/z \) is a bijection from \( \{ z : |z - 1| > 1, |z - 2| < 2 \} \) to the strip \( \{ z : 1 < \text{Re}(z) < 2 \} \).

2. Prove the Analytic Convergence Theorem: Let \( U \subset \mathbb{C} \) be an open, connected set and \( \{ f_n \} \) be a sequence of analytic functions on \( U \). If \( f_n \to f \) uniformly on every closed disk in \( U \) then \( f \) is analytic. Moreover, \( f'_n \to f' \) pointwise on \( U \) and uniformly on every closed disk in \( U \).

3. Fix \( R > 0 \). Show that there exists an integer \( n > 0 \) such that for all \( m \geq n \) the polynomial \( f_m(z) = \sum_{k=0}^{m} \frac{z^k}{k!} \) has no roots \( w \) with \( |w| < R \).

4. Prove that
\[
\int_0^\infty \frac{\sin(x)}{x} \, dx = \frac{\pi}{2}
\]

5. State and prove the Schwarz lemma.

6. Let \( f(z) = \sum_{n=0}^{\infty} a_n z^n \) have radius of convergence \( R < \infty \). Show that there exists a point \( w \) with \( |w| = R \) such that \( f \) can not be analytically continued to any open set which contains \( w \).

7. Can the function \( \text{Re}(z^2) \) be approximated uniformly on the unit circle \( \{ z : |z| = 1 \} \) by rational functions having only simple poles? Explain why or why not.

8. Let \( f \) be a non-constant, entire function such that \( f(1 - z) = 1 - f(z) \). Determine the image of \( f \).

9. Let \( \mathcal{F} \) be a family of holomorphic functions on the open unit disk \( \Delta \). Suppose that \( \mathcal{F}' = \{ f' \mid f \in \mathcal{F} \} \) is a normal family and there exists a point \( p \in \Delta \) such that \( \{ f(p) \mid f \in \mathcal{F} \} \) is bounded. Is \( \mathcal{F} \) a normal family?

10. Let \( U \) be a connected, open set and \( \{ a_n \} \) be a sequence of distinct points in \( U \) which do not have a limit point in \( U \). Fix an integer \( k \geq 0 \). Does there exist an analytic function \( f \) on \( U \) with prescribed values \( f(a_n), \ldots, f^{(k)}(a_n) \) for each \( n \)?