Stabilization of parabolic equation and Navier-Stokes system by feedback control

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We will consider a parabolic equation or Navier-Stokes system

$$\partial_t v(t) + Av(t) + B(v(t)) = f + Cu(t) \qquad v(0) = v_0 \tag{1}$$

where, A, C are appropriate linear operators and B is quadratic one, f, v_0 are given and $v(t), u(t)$ are unknown functions, where u is a control. Let $\hat{v}$ be a steady-state solution of (1), i.e. $A\hat{v} + B(\hat{v}) = f$.

The stabilization problem is formulated as follows: given $\alpha > 0$ find a control $u(t)$ such that the solution $v(t)$ of problem (1) satisfies the estimate

$$\|v(t)\| \leq ce^{-\alpha t} \quad \text{as} \quad t \to \infty \tag{2}$$

with suitable norm $\|\cdot\|$. In addition the control $u(t)$ should be feedback i.e. it has to react on perturbations of $v(t)$ damping them.

The stabilization problem is called local if it is assumed additionally that $\|\hat{v} - v_0\|$ is small enough, and otherwise it is called global.

Local stabilization problem by feedback control for Navier-Stokes system was actively studied by several authors during last 15 years. The first part of these lectures will be devoted to presentation of this problem solution.

The global stabilization problem is not solved yet completely but its essential part has been made already. In the second part of the lectures the results obtained to this direction will be discussed.

About local stabilization one can see [1] and references therein. On investigations connected with global stabilization problem see [2],[3].

[1] A.V.Fursikov, A.V.Gorshkov *Certain questions of feedback stabilization for Navier-Stokes equations.*-Evolution equations and control theory (EECT), v.1, N1, 2012, p.109-140.

[2] A.V.Fursikov *On the Normal-type Parabolic System Corresponding to the three-dimensional Helmholtz System.*- AMS Transl.Series 2, v.232 (2014), 99-118.

[3] A.V.Fursikov *Stabilization of the simplest normal parabolic equation by starting control.* Communication on Pure and Applied Analysis, v.13,# 5,September (2014),1815-1854.