We denote by $S(n)$ the sum of the base 10 digits of a natural number $n$. For example, $S(2018) = 2 + 0 + 1 + 8 = 11$.

**Problem 1.** Find all positive integers $n$ such that $S(5^n) = 2^n$.

**Problem 2.** Compute $S(S(S(2018^{2018})))$.

**Problem 3.** Find all positive integers $n$ such that

$$n + S(n) + S(S(n)) + S(S(S(n))) = 2018$$

**Problem 4.** Prove the following inequalities for all natural numbers $m$ and $n$

a) $S(m + n) \leq S(m) + S(n)$;

b) $S(mn) \leq S(m)S(n)$.

**Problem 5.** Prove that for every natural number $n$ we have

a) $S(n) \leq 8S(8n)$;

b) $S(n) \leq 5S(5^n n)$.

**Problem 6.** Prove that if $1 \leq x \leq 10^n$, then $S(x(10^n - 1)) = 9n$.

**Problem 7.** Find $S(9 \cdot 99 \cdot 9999 \cdot \ldots \cdot 99 \ldots 99)$, where each factor has twice as many digits as the previous one.

**Problem 8.** Prove that for every positive integer $n$ there exists a positive integer $x$ such that $x + S(x) = n$ or $x + S(x) = n + 1$.

**Problem 9.** Prove that there exist 50 pairwise distinct positive integers $n$ for which the value $n + S(n)$ is the same.

**Problem 10.** Does there exist $n$ such that $S(n) = 1000$ and $S(n^2) = 1000^2$?

**Problem 11.**

a) Does there exist $n$ such that

i) $S(n^2) = 2018$?

ii) $S(n^2) = 2017$?

b) Describe all $k$ for which there exists $n$ such that $S(n^2) = k$. 
