2020 Power Team Rules

- Each power team entry must have a cover sheet (**typed**). The cover sheet must contain the full name of the school (don't abbreviate), the coach's name, contact email, physical address (so that we know where to send the prizes in case your team wins), the team's name, and the names of each team member. For example, if a school has 3 power team entries, then there should be three different team names. The following is an example of an acceptable title page:

  2020 Power Team Entry
  XXXX School, Team 1
  Coach: Ms. Wizard
  Contact email:
  Address:
  Team Members:
  Jane Doe
  John Smith
  etc.

- Team participants are not allowed to consult with anyone but their teammates. Participants are not allowed to look on the web for any information regarding the power team exam, nor are they allowed to search books or other reference materials.
- Team entries are expected to be neat and legible. If not, they face the possibility of being disqualified by the judges.
- **Submissions:** This year, all submissions will be online and should be done no later than 9:00 AM on the day of the contest (Saturday, October 24). Send your solution to the email `hsmc@math.tamu.edu` and make your team's name as your email's subject line. Make sure the solution file is in PDF format (if your solution is handwritten, scan it first and save it as a **single** PDF file).

In case you need assistance with the exam submission, contact Oksana Shatalov (`shatalov@math.tamu.edu`).

GOOD LUCK!

2020 Power Team
Texas A&M High School Mathematics Contest
October 2020

In the following series of problems there is one or several particles of some masses on the real line at any moment t of time (time takes only integer values). If at moment t there is a particle of mass m at coordinate x, then (except for some special cases mentioned in the problems) the next moment $t+1$ it splits into two particles of mass $m/2$ each: one at coordinate $x+1$, and one at coordinate $x-1$ (all particles split simultaneously). If two particles of masses m_1 and m_2 meet at the same point of the line, then they merge and we get one particle at that point of mass $m_1 + m_2$.

For example, if at moment t there are particles of masses m_1, m_2, m_3, m_4 at coordinates $1, 2, 3, 4$, respectively, and no particles at other coordinates, then the next moment there are particles of masses $m_1/2, m_2/2, (m_1+m_3)/2, (m_2+m_4)/2, m_3/2, m_4/2$ at coordinates $0, 1, 2, 3, 4, 5$, respectively.

Problem 1. Suppose that at the initial moment $t = 0$ we have one particle of mass 1 at coordinate 0. Find the masses and coordinates of all particles at the moment $t > 0$.

Problem 2. How will the answer to Problem 1 change if we have an absorbing screen at coordinate $k > 0$: every particle that reaches that point is annihilated.

Problem 3. Find the answer when there are two absorbing screens at coordinates $k > 0$ and $l < 0$.

Problem 4. What if we have a reflective screen at coordinate $k > 0$: if a particle is at coordinate k at moment t, then it doesn't split into two particles, but moves instead to coordinate $k-1$ at moment $t+1$ without changing its mass.

Problem 5. Suppose now that we have a semi-transparent membrane at coordinate $k > 0$: if a particle of mass m is at coordinate k at moment t, then it is split into two particles next moment $t+1$: a particle of mass pm at coordinate $k-1$ and a particle of mass qm at coordinate $k+1$, where p, q are positive constants such that $p+q = 1$.

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In the following problems, we are looking for configurations of points inside a square room that stay as far from one another as possible.

Let Q be a unit square in the Euclidean plane (that is, a square with sides of length 1). Suppose S is a finite set of points inside the square Q (some points may lie on the boundary of Q). We denote by $sd(S)$ the minimal distance between distinct points in the set S. For any integer $n \geq 2$, let d_n be the maximal value of $sd(S)$ over all sets S of n points. A set S of n points inside the square Q is called an **optimal configuration** if $sd(S) = d_n$.

Problem 6. Find the optimal configurations of n points and find d_n for $n = 2, 4$, and 5. Prove that they are optimal.

Problem 7. The same for $n = 3$.

Problem 8. Find d_n and an optimal configurations of $n = 6$ and 8 points. (A rigorous proof is not required.)

Problem 9. Introduce Cartesian coordinates such that the vertices of the square are $(0,0), (1,0), (0,1)$, and $(1,1)$. For $k \geq 1$, let S_k be the set of points with coordinates of the form $\left(\frac{m_1}{k}, \frac{m_2}{k}\right)$ for $0 \leq m_1, m_2 \leq k$. Prove that for all sufficiently large k the configuration S_k is not optimal.