Texas A&M University
MPE 1 Math Placement Exam
Solutions Practice Problems

1. $\left(\dfrac{14}{3+\sqrt{2}}\right)\left(\dfrac{3-\sqrt{2}}{3-\sqrt{2}}\right)=\dfrac{42-14\sqrt{2}}{9-2}=\dfrac{7(6-2\sqrt{2})}{7}=6-2\sqrt{2}$

2. $\dfrac{x+2a-3}{x+a}-\dfrac{x+6}{2x}=\left(\dfrac{x+2a-3}{x+a}\right)\left(\dfrac{2x}{2x}\right)-\left(\dfrac{x+6}{2x}\right)\left(\dfrac{x+a}{x+a}\right)=\dfrac{(x+2a-3)(2x)-(x+6)(x+a)}{(x+a)(2x)}=$
$\dfrac{2x^2+4xa-6x-x^2-6x-xa-6a}{(x+a)(2x)}=\dfrac{x^2+3xa-12x-6a}{(x+a)(2x)}$

3. $\dfrac{6x^2+11xy-10y^2}{3x^2+10xy-8y^2}=\dfrac{(3x-2y)(2x+5y)}{(3x-2y)(x+4y)}=\dfrac{2x+5y}{x+4y}$

4. $\dfrac{(x^{-4}y^{2/5})^{-3/4}}{x^{2/3}y^{-5/6}}=\dfrac{x^{12/4}y^{-6/20}}{x^{2/3}y^{-5/6}}=x^{3-2/3}y^{-6/20--5/6}=x^{7/3}y^{32/60}=x^{7/3}y^{8/15}$

5. $5(x-7)-13(x-7)-6=0$
$(x-7)(5-13)-6=0$
$(x-7)(-8)-6=0$
$-8x+56-6=0$
$-8x+50=0$
$-8x=-50$
$x=50/8$

6. Multiple $-2x+4y=12$ by 3 and $3x-5y=-3$ by 2. This gives
$$\begin{aligned}-6x+12y&=36\\ 6x-10y&=-6.\end{aligned}$$
Add these two equations and we get $2y=30 \implies y=15$. Substituting $y=15$ into $3x-5y=-3$ gives $x=24$. So the point that satisfies both equations is $(24,15)$ and the value of $x+y$ is $24+15=39$.

7. Let x be the amount invested at $5\frac{1}{2}\%$ and y be the amount invested at $6\frac{3}{4}\%$. The resulting system of equations is
$$\begin{aligned}x+y&=10,000\\ 0.055x+0.0675y&=650.\end{aligned}$$
Solve the system of equations to find x and y.
$x=10,000-y \implies 0.055(10,000-y)+0.0675y=650 \implies 550-0.055y+0.0675y=650 \implies 0.0125y=100 \implies y=8000$. Since $x+y=10,000$ we know $x=2000$ and thus \$6,000 more is invested in $6\frac{3}{4}\%$.

8. Since $2ax+3by=7c \implies 3by=7c-2ax \implies y=\dfrac{7c-2ax}{3b}=\dfrac{-2a}{3b}x+\dfrac{7c}{3b}$. If x decreases by 10 then we know $y=\dfrac{-2a}{3b}(x-10)+\dfrac{7c}{3b}=\dfrac{-2ax}{3b}+\dfrac{20a}{3b}+\dfrac{7c}{3b}$, so y will increase by $\dfrac{20a}{3b}$.

9. $\dfrac{4k-6-16}{2k+3+2}=0 \implies \dfrac{4k-22}{2k+5}=0 \implies 4k-22=0 \implies k=22/4=5.5.$

10. (a) $V = (10-2x)(6-2x)x = 4x^3 - 32x^2 + 60x$

(b) $SA = 2x(6-2x)+2x(10-2x)+(6-2x)(10-2x) = 12x-4x^2+20x-4x^2+60-32x+4x^2 = -4x^2+60$

11. $\dfrac{5x+2}{x-10} \geq 3 \implies \dfrac{5x+2}{x-10} - 3 \geq 0 \implies \dfrac{5x+2-3(x-10)}{x-10} \geq 0 \implies \dfrac{5x+2-3x+30}{x-10} \geq 0 \implies \dfrac{2x+32}{x-10} \geq 0$. The expresstion $\dfrac{2x+32}{x-10}$ will be greater than or equal to zero on $(-\infty, -16] \cup (10, \infty)$.

12. To find the domain of $f(x) = \dfrac{\sqrt{x^2-3x-4}}{6x^2-54}$ we know $x^2-3x-4 \geq 0$ and $6x^2-54 \neq 0$. $x^2-3x-4 \geq 0$ for $(-\infty,-1]\cup[4,\infty)$ and $6x^2-54 \neq 0$ for $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$. So the domain of $f(x)$ is $(-\infty,-3)\cup(-3,-1]\cup[4,\infty)$.

13. For the function $\dfrac{2x^2+13}{(x+1)(x-1)}$ on $x < 0$ the domain is $(-\infty,-1)\cup(-1,0)$. For the function $\dfrac{5x-26}{x+2}$ on $x \geq 0$ the domain is $[0,\infty)$. So, for the function $f(x)$ the domain is $(-\infty,-1)\cup(-1,\infty)$.

14. $f(x) = \dfrac{6x^2-7x-5}{4x^2-12x-7} = \dfrac{(2x+1)(3x-5)}{(2x+1)(2x-7)} = \dfrac{3x-5}{2x-7}$, so the x-intercept is where $3x-5=0 \implies x = 5/3$.

15. $f(x) = \dfrac{6x^2-7x-5}{4x^2-12x-7} = \dfrac{(2x+1)(3x-5)}{(2x+1)(2x-7)} = \dfrac{3x-5}{2x-7}$, so the vertical asymptote is where $2x-7 = 0 \implies x = 7/2$. The horizontal asymptote is at $y = \dfrac{6}{4} = \dfrac{3}{2}$.

16. To find the x-interceps solve $f(x) = 0 \implies x(x+3)(x-3) = 0 \implies x = 0, \pm 3$. To find the y-intercepts, substitute $x = 0 \implies 0^3 - 9(0) = 0$. Hence the y-intercept is $y = 0$.

17. (a) We must have $-x^2-4x+5 \geq 0$ for $f(x)$ to be defined. Thus $(-x+1)(x+5) \geq 0$ which occurs on $[-5,1]$. Thus the domain of $f(x)$ is $[-5,1]$.

(b) We must have $4t-3 > 0$ for $g(t)$ to be defined. Thus, the domain for $g(t)$ is $\left(\dfrac{3}{4},\infty\right)$.

(c) We must have $x^3+3x^2-x-3 \neq 0$ for $h(x)$ to be defined. $x^3+3x^2-x-3 \neq 0 \implies x^2(x+3)-(x+3) \neq 0 \implies (x+3)(x^2-1) \neq 0 \implies (x+3)(x-1)(x+1) \neq 0 \implies x \neq \pm 1$ and $x \neq -3$. The the domain of $h(x)$ is $(-\infty,-3)\cup(-3,-1)\cup(-1,1)\cup(1,\infty)$.

18. $\left(\dfrac{2}{\sqrt{x^5}}\right)\left(\sqrt[3]{4x}\right) = \left(2x^{-5/2}\right)(4x)^{1/3}$

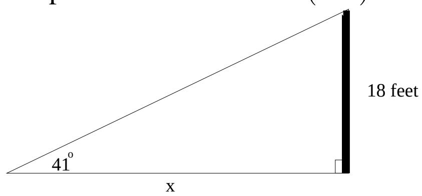
$= \left(2x^{-5/2}\right)(4^{1/3}\cdot x^{1/3}) = \left(2x^{-5/2}\right)\left((2^2)^{1/3}x^{1/3}\right) = \left(2x^{-5/2}\right)\left(2^{2/3}x^{1/3}\right) = \dfrac{2^{5/3}}{x^{13/6}}$

19. The transformed graph is $g(x) = \dfrac{1}{2}(x-4)^2 + 10$.

20. The domain of $\log(x+2)$ is $(-2,\infty)$. The domain of $\log(x-1)$ is $(1,\infty)$. So, the domain of $\log(x+2)+\log(x-1)$ is $(1,\infty)$. Now, use properties of logarithms to sovle the equation.
$\log(x+2)+\log(x-1) = 1 \implies \log[(x+2)(x-1)] = 1 \implies 10^1 = (x+2)(x-1) \implies 10 = x^2+x-2 \implies 0 = x^2+x-12 \implies 0 = (x+4)(x-3) \implies x = -4, 3$. Since $x = -4$ is not in the domain, $x = 3$ is the only solution.

21. $3x^2(4x^2+1)^8 + 64x^4(4x^2+1)^7 = x^2(4x^2+1)^7\left(3(4x^2+1)+64x^2\right)$
$= x^2(4x^2+1)^7(12x^2+3+64x^2) = x^2(4x^2+1)^7(76x^2+3)$

22. Refer to the figure below. We know that $\tan(41^\circ) = \dfrac{18}{x}$. So, $x = \dfrac{18}{\tan 41^\circ}$ or $x = 18\cot(41^\circ)$. So, the person must be $18\cot(41^\circ)$ feet from the base of the pole.



23. $f \circ g = \dfrac{\frac{2}{x}}{\frac{2}{x}+1} = \dfrac{\frac{2}{x}}{\frac{2}{x}+\frac{x}{x}} = \dfrac{\frac{2}{x}}{\frac{2+x}{x}} = \dfrac{2}{x}\cdot\dfrac{x}{2+x} = \dfrac{2}{2+x}$.

24. $\dfrac{8}{x+1} - \left(\dfrac{y}{z+2} \div \dfrac{y-4}{w}\right) = \dfrac{8}{x+1} - \left(\dfrac{y}{z+2}\cdot\dfrac{w}{y-4}\right) = \dfrac{8}{x+1} - \left(\dfrac{yw}{(z+2)(y-4)}\right)$
$= \dfrac{8(z+2)(y-4) - yw(x+1)}{(x+1)(z+2)(y-4)} = \dfrac{8zy - 32z + 16y - 64 - ywx - yw}{(x+1)(z+2)(y-4)}$

25. First factor the equation: $e^{2x} - 2e^x - 3 = (e^x)^2 - 2e^x - 3 = (e^x-3)(e^x+1)$. Solving $(e^x-3)(e^x+1) = 0 \implies e^x - 3 = 0$ or $e^x + 1 = 0 \implies e^x = 3$ or $e^x = -1$, but e^x will never be negative, so the only solution is $e^x = 3 \implies x = \ln(3)$.

26. Using point slope equation we get $y - 1 = 7(x-5) \implies y = 7x - 35 + 1 \implies y = 7x - 34$. Use this to find y when $x = -4$ gives $y = 7(-4) - 34 \implies y = -62$.

27. $\dfrac{f(2+h)-f(2)}{h} = \dfrac{\sqrt{2+h+4}-\sqrt{2+4}}{h} = \left(\dfrac{\sqrt{6+h}-\sqrt{6}}{h}\right)\cdot\left(\dfrac{\sqrt{6+h}+\sqrt{6}}{\sqrt{6+h}+\sqrt{6}}\right) = \dfrac{6+h-6}{h\left(\sqrt{6+h}+\sqrt{6}\right)}$
$= \dfrac{1}{\sqrt{6+h}+\sqrt{6}}$

28. $\dfrac{(x^2y^4)^5(x^3y)^{-3}}{xy} = \dfrac{x^{10}y^{20}x^{-9}y^{-3}}{xy} = \dfrac{xy^{17}}{xy} = y^{16}$.

29. $\sqrt[3]{(a^3b)}\sqrt[3]{64a^4b^2} = \sqrt[3]{64a^7b^3} = 4a^2b(\sqrt[3]{a})$

30. $\dfrac{x^2}{x^2-x-2} - \dfrac{4}{x^2+x-6} + \dfrac{x}{x^2+4x+3} = \dfrac{x^2}{(x-2)(x+1)} - \dfrac{4}{(x+3)(x-2)} + \dfrac{x}{(x+1)(x+3)}$
$= \left(\dfrac{x+3}{x+3}\right)\left(\dfrac{x^2}{(x-2)(x+1)}\right) - \left(\dfrac{x+1}{x+1}\right)\left(\dfrac{4}{(x+3)(x-2)}\right) + \left(\dfrac{x-2}{x-2}\right)\left(\dfrac{x}{(x+1)(x+3)}\right)$
$\dfrac{x^3+3x^2-4x-4+x^2-2x}{(x+3)(x-2)(x+1)} = \dfrac{x^3+4x^2-6x-4}{(x+3)(x-2)(x+1)}$

31. Factor and simplify: $f(x) = \dfrac{3x^2-14x-5}{4x^2-17x-15} = \dfrac{(3x+1)(x-5)}{(4x+3)(x-5)} = \dfrac{3x+1}{4x+3}$. This yields a zero of $x = -\dfrac{1}{3}$, a vertical asymptote of $x = -\dfrac{3}{4}$, and a horizontal asymptote $y = \dfrac{3}{4}$.

32. Using the identity $\cos^2\theta + \sin^2\theta = 1$, we find that $\cos^2\theta = 1 - \sin^2\theta = 1 - \dfrac{1}{49} = \dfrac{48}{49}$. This gives us $\cos\theta = \pm\sqrt{\dfrac{48}{49}} = \pm\dfrac{\sqrt{48}}{7} = \pm\dfrac{4\sqrt{3}}{7}$. Since θ is in quadrant II, $\cos\theta < 0$, hence $\cos\theta = -\dfrac{4\sqrt{3}}{7}$.

33. We know $\ln(ab) = \ln a + \ln b$; $\ln\left(\dfrac{a}{b}\right) = \ln a - \ln b$, and $\ln a^b = b\ln a$. Using these properties, we obtain $\ln\left(\dfrac{\sqrt{x}y^5}{(z+1)^4}\right) = \ln\left(\sqrt{x}y^5\right) - \ln(z+1)^4 = \ln(x^{1/2}y^5 - 4\ln(z+1) = \ln x^{1/2} + \ln y^5 - 4\ln(z+1) = \dfrac{1}{2}\ln x + 5\ln y - 4\ln(z+1)$.

34. $\sec \dfrac{2\pi}{3} - \tan \dfrac{\pi}{6} = -2 - \dfrac{1}{\sqrt{3}}$

35. We begin with a rectangle of area 20 in^2. If we increase the length by 8%, then our new length is 5.4 inches. This gives a new area of 21.6 in^2. This give a total change in area of 1.6 in^2.

36. $f(2) = (2)^3 + 1 = 9$ and $f(-3) = 2(-3)^2 - 3 = 18 - 3 = 15$, so $f(2) - f(-3) = 9 - 15 = -6$.

37. Use the identity $\cos^2\theta = 1 - \sin^2\theta$ to get $\dfrac{\cos^2(\theta)}{1+\sin(\theta)} = \dfrac{1-\sin^2\theta}{1+\sin\theta} = \dfrac{(1+\sin\theta)(1-\sin\theta)}{1+\sin\theta} = 1 - \sin\theta$.

38. $\log_4\left(\dfrac{1}{\sqrt[3]{16}}\right) = \log_4\left(\dfrac{1}{\sqrt[3]{4^2}}\right) = \log_4\left(\dfrac{1}{4^{2/3}}\right) = \log_4 4^{-2/3} = -2/3$

39. $\dfrac{\frac{1}{a} - b}{\frac{1}{b^3} + a} = \dfrac{(\frac{1}{a} - b)(ab^3)}{(\frac{1}{b^3} + a)(ab^3)} = \dfrac{b^3 - ab^4}{a + a^2b^3}$

40. Use the exponential growth model $y(t) = y_0e^{kt}$, where $y(t)$ is the size of the population at time t and y_0 is the initial size of the population. We know that $y_0 = 1200$, hence $y(t) = 1200e^{kt}$. Since the population doubles every day, we know $2400 = 1200e^{k\cdot 24} \implies 2 = e^{24k} \implies \ln 2 = 24k \implies k = \dfrac{1}{24}\ln 2 = \ln 2^{1/24}$. Thus, $y(t) = 1200e^{t\ln 2^{1/24}} = 1200e^{\ln 2^{t/24}} = 1200 \cdot 2^{t/24}$ (note t is in hours). Now solve for t when $y(t) = 10000 \implies 10000 = 1200 \cdot 2^{t/24} \implies \dfrac{10000}{1200} = 2^{t/24} \implies \dfrac{25}{3} = 2^{t/24} \implies \log_2\left(\dfrac{25}{3}\right) = \dfrac{t}{24} \implies t = 24\log_2\left(\dfrac{25}{3}\right)$. So, it will take $24\log_2\left(\dfrac{25}{3}\right)$ hours for the culture to reach 10,000 bacteria.