2020 Texas A&M REU Miniconference

July 20–21, via Zoom

SCHEDULE

MON. July 20	**10:00–10:20**	Patterns arising in the Kernel of Generalized Dedekind Sums (part 1)	Juan Ramirez
	10:25–10:45	Patterns arising in the Kernel of Generalized Dedekind Sums (part 2)	Eveuilynn Nguyen
	10:50–11:10	Characterizing Codes with Three Maximal Codewords	Clare Spinner
	11:10–14:00	*Break (lunch)*	
	14:00–14:20	Identifiability of Linear Compartment Models: The effects of model operations	Kate Johnston
	14:25–15:05	Parallel SSEP and a Casimir Element of $\mathfrak{so}_{2n}$	Andy Park and Mark Landry
	15:10–15:20	*Break*	
	15:20–15:40	A Central Element of the Quantum Group $U_q(\mathfrak{so}_{2n})$	Andrew Lin

TUE. July 21	**10:00–10:20**	Algebraic signatures for a non-local obstruction and sunflowers	Joseph Lent
	10:25–10:45	Degenerate and non-degenerate embedding dimensions of neural codes	Patrick Chan
	10:50–11:00	*Break*	
	11:00–11:20	Bounds for Coefficients of the $f(q)$ Mock Theta Function and Applications for Partition Ranks (part 1)	Eric Zhu
	11:25–11:45	Bounds for Coefficients of the $f(q)$ Mock Theta Function and Applications for Partition Ranks (part 2)	Kevin Gomez

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ABSTRACTS

(In order of appearance)

Patterns arising in the Kernel of Generalized Dedekind Sums

Juan Ramirez and Eveuilynn Nguyen

We study a generalized version of the Dedekind sum associated with a pair of non-trivial primitive Dirichlet characters χ_1, χ_2 denoted $S_{\chi_1,\chi_2}(\gamma)$. Focusing in on the crossed homomorphism $\varphi : \Gamma_0(q_1q_2) \to \mathbb{C}$, we expand the work of Dillon and Gaston to provide interesting symmetries of the Kernel of the crossed homomorphism φ over the congruence subgroup $\Gamma_0(q_1q_2)$.

Characterizing Codes with Three Maximal Codewords

Clare Spinner

Neural codes, represented as collections of binary strings called codewords, are used to encode neural activity. A code is called convex if its codewords are represented as an arrangement of convex sets in Euclidean space. It is of particular interest to the mathematical community to determine which codes can be represented by open or closed convex sets. We can determine the convexity of a code if it has at most two maximal codewords, but the cases where there are more maximal codewords remain unsolved. We present a complete characterization of codes with exactly three maximal codewords and their embedding dimensions.

Identifiability of Linear Compartment Models: The effects of model operations

Kate Johnston

Linear compartment models, used to represent the transfer of substances in a system, have applications in a variety of fields including ecology, physiology, and pharmacokinectics. The study of identifiability seeks to determine which unknown transfer rates in a given linear compartment model can be recovered from input-output data. In this presentation we address the effects of model operations on the identifiability of the model. More specifically, we focus on the effects of the addition and removal of a leak, and the deletion of dividing edges to show in what cases the identifiability of the model is preserved.

Parallel SSEP and a Casimir Element of $\mathfrak{so}_{2n}$

Andy Park and Mark Landry

Previous research has demonstrated that one can obtain the generator of a Markov process through the choice of a Casimir Element and representation of a chosen Lie algebra. One example is $\mathfrak{sl}_2$, which leads to the generator for the Symmetric Simple Exclusion Process (SSEP). In this presentation, we demonstrate how a representation of a Casimir element from $\mathfrak{so}_{2n}$ leads to a generator matrix of a rigorously defined type of Parallel SSEP, a system of distinct SSEP's in parallel to each other, which has properties relating to class, mass, and mass-preserving transformations that allow concurrent moves elsewhere in the system.

A Central Element of the Quantum Group $U_q(\mathfrak{so}_{2n})$

Andrew Lin

We construct an explicit central element of the quantum groups $U_q(\mathfrak{so}_6)$ and $U_q(\mathfrak{so}_8)$ and provide progress towards the general case $U_q(\mathfrak{so}_{2n})$. We use this central element to construct a Markov generator matrix and describe the resulting asymmetric particle process.

Algebraic signatures for a non-local obstruction and sunflowers
Joseph Lent
Neural codes dictate intersection relationships for the receptive fields they generate. According to experimentation, these sets should be convex. However, local and non-local obstructions to convexity exist. While local obstructions have been studied extensively, non-local obstructions have not. We show that algebraic signatures in the canonical form of the neural ideal reveal these non-local obstructions. We also study the neural ideals of Sunflowers, S_n, and provide the method for constructing a closed convex realization for S_n.

Degenerate and non-degenerate embedding dimensions of neural codes
Patrick Chan
Neural codes are a collection of binary strings that represent the possible combinations of neurons firing within a set of receptive fields. Specifically in the case of place cells, these receptive fields are correlated to convex areas in space. Thus, when dealing with neural codes in relation to place cells, there is a desire for codes to be able to be represented with convex receptive fields. From this desire comes the need to understand the embedding dimensions in which codes require for convexity. In this presentation I will show our finding thus far, explaining how we classify neural code realizations and the relationships between the open and closed embedding dimension. Specifically, I'll present the equivalent relation between the open and closed non-degenerate embedding dimensions, how non-degenerate embedding dimensions act as an upper bound for the general embedding dimensions, and lastly present a hypothesis and example(s) of the open and closed degenerate embedding dimension relation.

Bounds for Coefficients of the $f(q)$ Mock Theta Function and Applications for Partition Ranks
Eric Zhu and Kevin Gomez
Ramanujan's mock theta function $f(q)$ is a q-hypergeometric series with great importance to number theorists in the analysis of partition ranks modulo two. Recent work due to Brunier and Schwagenscheidt expresses the Fourier coefficients of $f(q)$ as finite algebraic formulas. We discuss the techniques used to obtain effective bounds for the Fourier coefficients from these formulas and the application of our bounds to resolve a conjecture of Hou and Jagadeesan on the convexity of the modulo two rank counting functions.