

Inoculation Strategies for Polio: Modeling the Effects of a Growing Population on Public Health Outcomes

Meredith McCormack-Mager

July 23, 2014

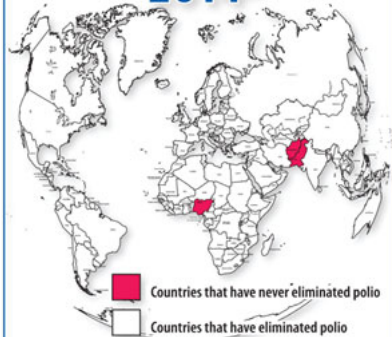




1988



2014*



*As of April 29, 2014

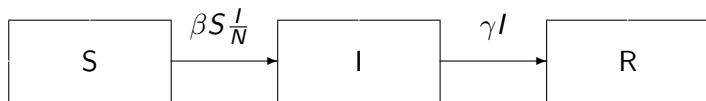


Figure : Basic SIR Model

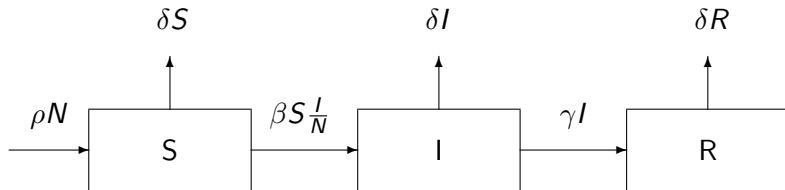


Figure : Basic SIR Model with Non-Constant Population

Model

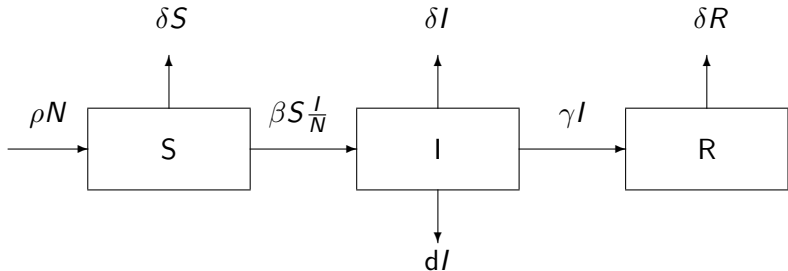


Figure : Basic SIR Model with Non-Constant Population and Death from Disease

Epidemic

A rapid spread, growth, or development.

Endemic

Maintained in a population without external inputs.

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A rapid spread, growth, or development.

Endemic

Maintained in a population without external inputs.

Epidemic

When is $I'(t) > 0$?

$$I'(t) = \beta S \frac{I}{N} - \delta I - dI - \gamma I > 0$$

$$R_0 := \frac{\beta}{\delta + d + \gamma} > 1$$

$$R_{0\text{constant}} := \frac{\beta}{\gamma} > 1$$

Epidemic

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Endemic

What is the end behavior of $I(t)$?

$$\frac{d\left(\frac{I}{N}\right)}{d\left(\frac{S}{N}\right)} = \frac{\left(\frac{I}{N}\right)'(t)}{\left(\frac{S}{N}\right)'(t)} > 0$$

Endemic

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$$\frac{d\left(\frac{I}{N}\right)}{d\left(\frac{S}{N}\right)} = \frac{\left(\frac{I}{N}\right)'(t)}{\left(\frac{S}{N}\right)'(t)} > 0$$

Model: Vaccination

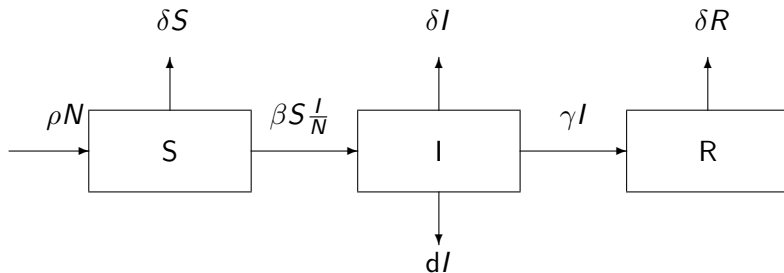


Figure : Basic SIR Model with Non-Constant Population, Death from Disease, and Vaccination of Newborns

Model: Vaccination

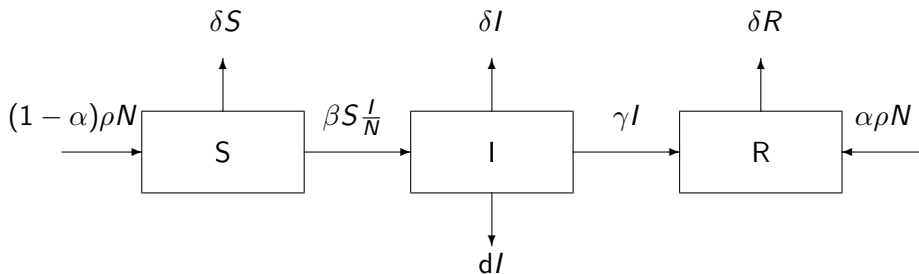
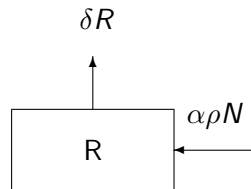
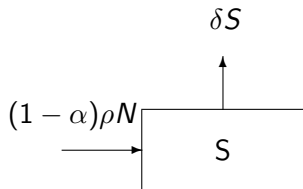
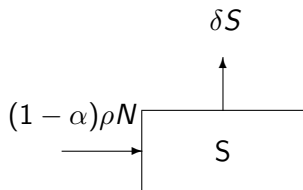


Figure : Basic SIR Model with Non-Constant Population, Death from Disease, and Vaccination of Newborns

Evaluating the Disease Free State

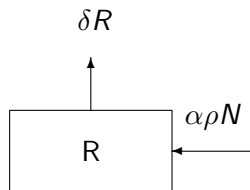


Evaluating the Disease Free State



$$\frac{S}{N}(t) = (1 - \alpha)$$

$$\frac{I}{N}(t) = 0$$



$$\frac{R}{N}(t) = \alpha$$

Preventing an Epidemic

When is $I'(t) < 0$?

$$R_0 := \frac{\beta \frac{S}{N}}{\delta + d + \gamma} < 1$$

$$\frac{\beta(1 - \alpha)}{\delta + d + \gamma} < 1$$

$$1 - \frac{\delta + d + \gamma}{\beta} < \alpha$$

Model: Split Age Classes

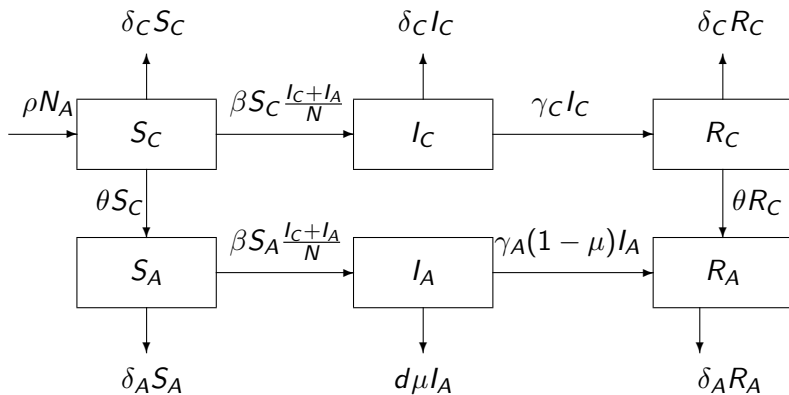


Figure : Split Age Class SIR Model

Model: Vaccinating 1-4 Year-Olds

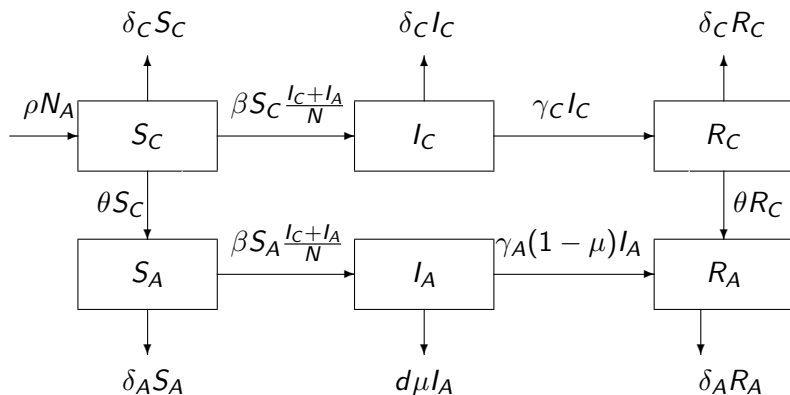


Figure : Split Age Class SIR Model For 1-5 Year-Old Vaccination Strategy

Model: Vaccinating 1-4 Year-Olds

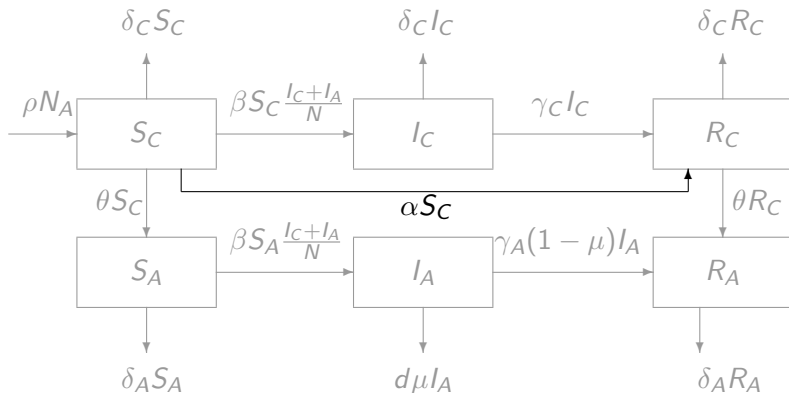


Figure : Split Age Class SIR Model For 1-5 Year-Old Vaccination Strategy

Model: Vaccinating 1-4 Year-Olds

$$R_0 := \frac{\beta \frac{S_C(t)}{N(t)}}{\delta_C + \gamma_C} + \frac{\beta \frac{S_A(t)}{N(t)}}{d\mu + \gamma_A(1 - \mu)} < 1$$

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$$\delta_A(\delta_C + \theta) > \rho\theta$$

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$$\delta_A(\delta_C + \theta) > \rho\theta$$

ρ = birth rate = 0.038

δ_C = child mortality rate = .124

θ = maturation rate = 0.25

δ_A = adult mortality rate = .013

Model: Vaccinating 1-4 Year-Olds

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$$\delta_A(\delta_C + \theta) > \rho\theta$$

ρ = birth rate = 0.038

δ_C = child mortality rate = .124

θ = maturation rate = 0.25

δ_A = adult mortality rate = .013

$$0.0049 \not> 0.0095$$

Model: Vaccinating Newborns (Age 0)

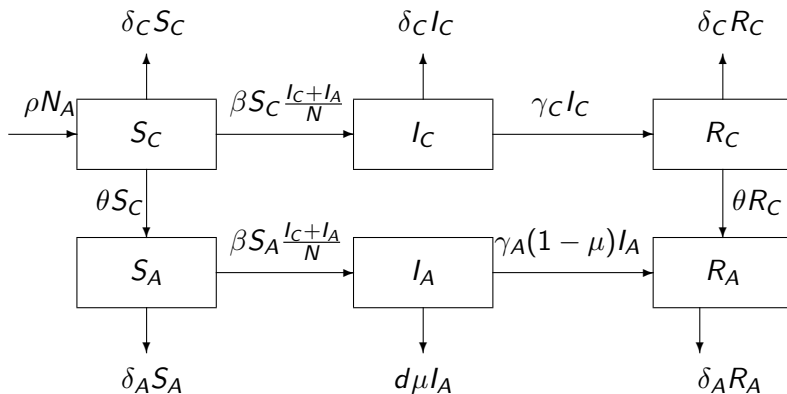


Figure : SIR Model For Newborn Vaccination Strategy

Model: Vaccinating Newborns (Age 0)

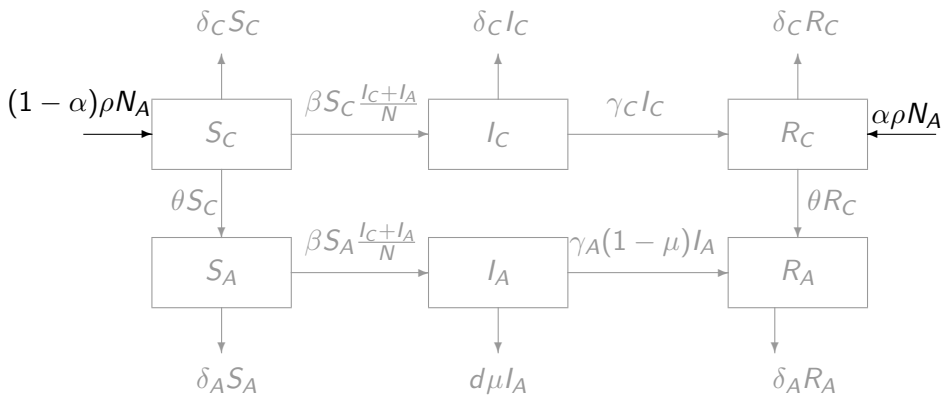


Figure : SIR Model For Newborn Vaccination Strategy

Model: Vaccinating Newborns (Age 0)

$$R_0 := \frac{\beta \frac{S_C(t)}{N(t)}}{\delta_C + \gamma_C} + \frac{\beta \frac{S_A(t)}{N(t)}}{d\mu + \gamma_A(1 - \mu)} < 1$$

Model: Vaccinating Newborns(Age 0)

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$$1 - \frac{\delta_A(\delta_C + \theta)}{\rho\theta} < 2\alpha$$

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$$0.244 < \alpha$$

Model: Vaccinating Newborns(Age 0)

$$R_0 := \frac{\beta \frac{S_C(t)}{N(t)}}{\delta_C + \gamma_C} + \frac{\beta \frac{S_A(t)}{N(t)}}{d\mu + \gamma_A(1 - \mu)} < 1$$

$$1 - \frac{\delta_A(\delta_C + \theta)}{\rho\theta} < 2\alpha$$

ρ = birth rate = 0.038

δ_C = child mortality rate = .124

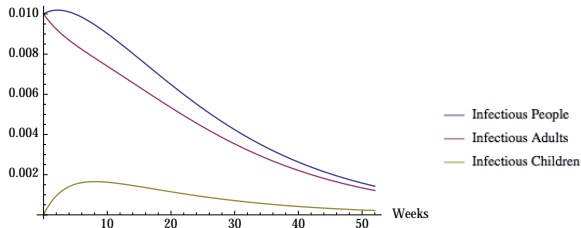
θ = maturation rate = 0.25

δ_A = adult mortality rate = .013

$$0.244 < \alpha$$

Minimum $\alpha = 0.78$

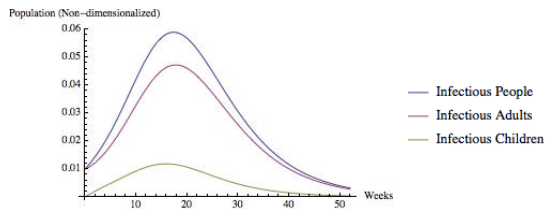
Population (Non-dimensionalized)



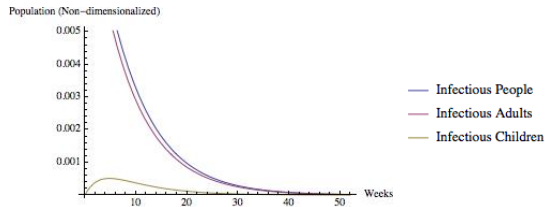
$\alpha = 0.78$

Minimum $\alpha = 0.78$

$\alpha = 0.6$



$\alpha = 0.9$



- ▶ Combination model for vaccination
- ▶ Continued analysis of age structures
- ▶ Proof of equilibrium state taking into consideration death from disease

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