2015 Texas A&M REU Miniconference

July 20–21, Blocker Building, Room 457

Schedule and Abstracts

SCHEDULE

MONDAY July 20

09:00–09:25	Breakfast snacks in Blocker 246	Bluebaker
09:30–09:55	The Asymptotic Distribution of Andrews' Smallest Parts Function	Josiah Banks and Kevin Sheng
10:00–10:25	The Number of Roots of Trinomials over Prime Fields	Zander Kelley
10:35–11:00	Zeros of the Modular Form $E_kE_l - E_{k+l}$	Polina Vulakh and Sarah Reitzes
11:05–11:30	An Infinite Family of Chemical Reaction Networks with Multiple Non-Degenerate Equilibria	Bryan Felix and Zev Woodstock
11:35–12:00	Analysis of an immunological subnetwork	Antoine Marc
12:00–13:00	Lunch in Blocker 246	Taz
13:00–13:25	Testing Thermodynamic Compliance of Chemical Reaction Networks Efficiently	Meredith McCormack-Mager
13:30–14:00	Extremal Trinomials over Quadratic Fields	Sean Owen

TUESDAY July 21

09:00–09:25	Incorporating HDL Interaction in an ODE model of Atherosclerosis	Diego Lopez
09:30–09:55	Polynomial System Solving Using Archimedean Tropical Varieties	Yasmina Marden and Hailey Stierneman
10:00–10:15	Group pictures!	Mitchell Physics Building Garden
10:20–10:45	Synchronizing Molecular Clocks via ATP: What can we predict?	Joseph Donnelly
10:50–11:15	Analysis of the Seasonality in Lyme Disease	Carlos Munoz
11:20–11:45	Finding and Interpreting Obstructions to Convexity in Neural Codes	Caitlin Lienkaemper
11:50–12:15	Refining Fewnomial Theory for Certain 2x2 Systems	Mark Stahl
12:15–	Lunch in Blocker 246	Bluebaker

ABSTRACTS

(In order of appearance)

The Asymptotic Distribution of Andrews' Smallest Parts Function
by Josiah Banks and Kevin Sheng
We present an asymptotic formula for Andrews' smallest parts function, which counts the number of smallest parts associated to partitions of a positive integer. An important role is played by the distribution of certain points in the complex upper half-plane called Heegner points; they are roots of binary quadratic forms of a given negative discriminant.

The Number of Roots of Trinomials over Prime Fields
by Alexander Kelley
Let $\#Z(f)$ denote the cardinality of the zero-set of a polynomial f over a finite field $\mathbb{F}_q$ with q a prime power. In their recent work, Cheng, Gao, Rojas, and Wan establish an upper bound for $\#Z(f)$ when f is a sparse polynomial. However, they observe that in the case of trinomials over $\mathbb{F}_p$ (with p a prime), their bound ($O(\sqrt{p})$) appears to be far from optimal. We present computational data that suggests that the number of roots of trinomials may grow as slowly as $O(\log p)$. Additionally, we investigate "typical" values for $\#Z(f)$. In particular, we conjecture that $\#Z(f)$ is Poisson-distributed when $\mathbb{F}_p$ and $deg(f)$ are fixed and the coefficients of f vary. Finally, we prove a weaker version of this statement under the Generalized Riemann Hypothesis.

Zeros of the Modular Form $E_kE_l - E_{k+l}$
by Sarah Reitzes and Polina Vulakh
The Eisenstein series of weight k is a modular form defined by $E_k(z) = \frac{1}{2}\sum\sum(cz+d)^{-k}$, where the sum is taken over all relatively prime integers c and d. We focus on the behavior of the Eisenstein serires in the intersection of the strip $-\frac{1}{2} \leq x \leq \frac{1}{2}$ with the region $|z| \geq 1$, called the fundamental domain. Rankin and Swinnerton-Dyer proved that all of the zeros of the Eisenstein series in the fundamental domain lie on the arc $|z| = 1$. In this study we consider the zeros of the weight $k+l$ modular form $E_kE_l - E_{k+l}$ in the fundamental domain. Based on numerical evidence, we conjecture that all of its zeros lie on the boundary. In the extreme case where $l = 4$, we conjecture that all of the zeros lie on the arc $|z| = 1$, while at the opposite extreme where $k = l$, we conjecture that all of the zeros lie on the lines $x = \pm\frac{1}{2}$. Using asymptotoic approximations, we have proved that for $l \in \{4, 6, 8\}$ all of the zeros lie on the arc $|z| = 1$ and that for $k = l$, almost all of the zeros lie on the lines $x = \pm\frac{1}{2}$.

An Infinite Family of Chemical Reaction Networks with Multiple Non-Degenerate Equilibria
by Bryan Felix and Zev Woodstock
Multistationary chemical networks are a subject of interest to scientists and mathematicians alike. While some criteria for multistationarity have been given, explicitly solving for these rates and steady-state concentrations is nontrivial. Nonetheless, for a specific family of systems K(m,n), we describe in this talk how to give closed forms for their rates and steady-state concentrations. We accomplish this via techniques developed by Craciun and Feinberg. Our results allow us to prove that the steady states are non-degenerate when n=3, thereby resolving one case of a conjecture of Joshi and Shiu. Additionally, we show for the first time that the method of Craciun and Feinberg can give rise to a degenerate steady state.

Analysis of an immunological subnetwork
by Antoine Marc
Immunology has always been a complex area of study involving many different pathways and cellular interactions. Mathematically, this can be represented by systems of ordinary differential equations. Though analysis of ODEs is simple at first, the complexity of the interactions that take place within a large network of cells, as is the case in Immunology, makes simple analysis much more challenging. However, it has been shown that a certain immunological network not only has an equilibrium value, but under certain conditions

admits multiple equilibrium values. This bistability is of importance to biologists as it indicates tolerance, a characteristic of the immune system that is being heavily studied in recent years. It is known that under certain hypotheses if a subnetwork of chemical reactions is bistable then the original, larger network of chemical reactions must also be bistable. In this presentation, I will analyze the subnetworks inside the already proven bistable network in order to gain insight on the details of the biology at hand.

Testing Thermodynamic Compliance of Chemical Reaction Networks Efficiently
by Meredith McCormack-Mager
Current algorithms for checking whether a chemical reaction network obeys the second law of thermodynamics are slow. This talk will demonstrate that using matroids to test thermodynamic feasibility is always exponential in the worst case. This talk introduces a new algorithm for determining thermodynamic viability of chemical reaction networks based on linear programming. This method runs in polynomial time, and has much promise to improve in complexity towards superlinear convergence as interior point methods improve.

Extremal Trinomials over Quadratic Fields
by Sean Owen
Over the real numbers, Descartes's Rule provides an optimal upper bound for the number of roots of a sparse polynomial. While an equally strong finite field analogue of this rule remains unknown, Bi, Cheng, and Rojas (2014) have recently discovered new upper bounds on root counts over $\mathbb{F}_{p^k}$, and Cheng, Gao, Rojas, and Wang (2015) showed that these bounds are near-optimal for many cases. However, sharp bounds have not been found for the case $k = 2$. We begin to fill this gap by presenting new bounds on the number of roots of trinomials over $\mathbb{F}_q$ with q the square of a prime p, with mild constraints on the exponents. Specifically, we reduce the problem to bounding the roots of trinomials of the form $1 + cx^{a_2} - (c + 1)x^{a_3}$, and, based on the disjointness of root sets of trinomials of this form, we prove an upper bound on their root counts. Using linear maps with large null spaces, we also construct extremal trinomials having this maximal number of roots. The result is an optimal upper bound of $\sqrt{q}$. Our methods offer several possible generalizations to fields of higher degree and sparse polynomials with more terms.

Incorporating HDL interaction in an ODE model of Atherosclerosis
by Diego Lopez
Atherosclerosis is a cardiovascular disease characterized by the build-up of fatty plaques in the intima of an arterial wall following inflammation of the lining of the artery, which slowly lead to occlusion of the arterial lumen and hardening of the artery walls. One third of Americans over the age of 35 die each year of atherosclerosis of the heart and half of men and women over the age of forty are reported to have some form of this illness. The medical community has long recognized that the concentration profile of low density lipoprotein (LDL) and high density lipoprotein (HDL) in the blood plasma directly correlate to the risk of developing atherosclerosis. In this talk, I go into the details of the physiology of atherosclerosis, propose a simple ODE model following some reasonable assumptions, and present equilibrium solutions of LDL concentration, HDL concentration, and other important components of the system.

Polynomial System Solving Using Archimedean Tropical Varieties
by Yasmina Marden and Hailey Stirneman
Suppose you know the location of three beacons and you want to find the position of an unknown point P. Suppose further that you know the angle from between each pair of beacons, as seen from P. Using the property of equiangular circles and chords, we create three distinct semi-algebraic curves that each contain the point P. We then find bivariate quadratic polynomials that define these curves and create four 2×2 systems of polynomial equations from two of the semi-algebraic curves and approximate the solutions to the four systems using tropical geometry. We show how tropical geometry can be used to derive binomial systems yielding start points good enough for Newton Iteration to quickly recover the point P. The motivation for our approach is an efficient method for solving general polynomials systems.

In particular, our method should be simple enough to be practically useful for 3D position estimation on radition-hardened processors present in spacecraft.

What Is The Role of ATP in Molecular Clock Synchronization?
by Joseph Donnelly
The environment produces repetitive, predictable stimuli. The sun sets routinely each night, and animals adapt automatically by establishing subconscious sleeping patterns. Physiological patterns arise from oscillators known as molecular clocks. These biochemical timekeepers are present in nearly all of an organisms cells. Molecular clock synchrony is crucial to prevent weakening of collective output. Recent investigation of brain cells in mice suggests a synchronizing role of ATP in the mammalian clock. The biochemical mechanism of synchronization via ATP remains unknown. Furthermore, instances in which ATP behaves as a signaling molecule are exceedingly rare. The Scheper Model, a system of two delay differential equations, is used to simulate the interaction of ATP with the mammalian clock. I demonstrate theoretical feasibility of the synchronization process, and offer a method of quantifying synchrony between mammalian clocks. Jointly, these techniques yield predictive power and the potential to intelligently manipulate synchrony among molecular clocks.

Analysis of the Seasonality in Lyme Disease
by Carlos Muñoz
After reviewing some background on Lyme disease, including its transmission and life-cycle, we describe an ODE model describing its spread.

Finding and Interpreting Obstructions to Convexity in Neural Codes
by Caitlin Lienkaemper
How does your brain keep track of your position in space? In a familiar environment, hippocampal neurons known as place cells become associated to regions of space known as their receptive fields. The firing patterns of place cells form a neural code. Given a set of receptive fields, it is very easy to determine the neural code which describes the place fields. Given a neural code, when is it possible to find a set of receptive fields which form a good cover and generate that neural code? It is known that if a neural code has certain local obstructions, there is no good cover which generates the code. It is not known whether or not the converse of this theorem is true. In this talk, we present work on this converse in some cases. Additionally, neural codes have previously only been classified as having local obstructions or not on up to four neurons. We pursue the case of five neurons, by first enumerating all simplicial complexes on five vertices and then describing the set of neural codes without local obstructions arising from each simplicial complex.

Refining Fewnomial Theory for Certain 2×2 Systems
by Mark Stahl
Finding the correct extension of Descartes' Rule to multivariate polynomial systems remains a distributioncult open problem. We focus on pairs of polynomials where the first has 3 terms and the second has m, and we say these systems are of type $(3, m)$. For these systems, recent work has determined that the maximum finite number of roots in the positive quadrant lies between $2m - 1$ and $\frac{2}{3}m^3 + 5m$. The techniques applied so far are variants of Rolle's Theorem and a forgotten result of Polya on the Wronskian of certain analytic functions. We add to these techniques by considering curvature, as a step to- ward establishing sharper upper bounds. We also build new extremal examples of minimal height.