Analyzing Methods to Determine Pairwise Correlations Between Neurons

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Neurons fire signals, known as *spikes*, in order to communicate with each other through an electrochemical process.

Each neuron has a corresponding spike train, which is a sequence of spikes over time.

In an attempt to explain relationships between neurons based on spike trains, Okun *et al.* discussed these complex individual neural activities and how they could be coordinated [3].

Found that neighboring neurons were correlated based on the firings of the overall population and found that this provided a compact summary of the population activity.
RMM with Coupling Terms: The Parameters

3 parameters extracted to determine pairwise correlations:

1. Row sums $s$: number of spikes of a neuron
2. Column sums $c$: number of all spikes at $time = i$
3. Inner product of each row and $c$, $d$: stPR

Example

<table>
<thead>
<tr>
<th>Neurons</th>
<th>Raster Plot</th>
<th>$s$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 1 0 1 0 0 1 1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1 0 1 0 1 1 0 0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>1 0 1 0 1 1 0 0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>c</td>
<td>2 1 2 1 2 2 1 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$M(s, c, d) = \{\text{set of matrices with prescribed } (s, c, d)\}$
First they generated a matrix satisfying $s$ and $c$ using Ryser’s algorithm [1].

Then the matrix was put in canonical form and a random spike exchange across neurons [2] was performed.

**Figure:** A representation of spike exchange across neurons [2]
Example cont’d

Here is a possible matrix that we may obtain from this process:

<table>
<thead>
<tr>
<th>Neurons</th>
<th>New Raster Plot</th>
<th>s*</th>
<th>d*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 0 0 0 0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 0 1 1 1 1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 1 0 0 0 0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>c*</td>
<td>2 2 2 2 1 1 1 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**RMM with Coupling Terms: \( d \) Constraint**

### Example cont’d

<table>
<thead>
<tr>
<th>Neurons</th>
<th>New Raster Plot</th>
<th>( s^* )</th>
<th>( d^* )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1 0 0 0 0</td>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 0 1 1 1 1</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 1 0 0 0 0</td>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>( c^* )</td>
<td>2 2 2 2 1 1 1 1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exchange the 0’s and the 1’s in the boxed sub-matrix above:

<table>
<thead>
<tr>
<th>Neurons</th>
<th>New Raster Plot</th>
<th>( s^* )</th>
<th>( d^* )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 0 1 0 0 0 0</td>
<td>4</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 1 0 1 1 1</td>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 1 0 0 0 0</td>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>( c^* )</td>
<td>2 2 2 2 1 1 1 1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Finally, the correlation between each pair of neurons from this new random matrix was computed using the Pearson correlation.

**Definition**

The *Pearson correlation* is a measure of strength of the linear relationship between two variables $x$ and $y$.

\[
\begin{align*}
    r &= \frac{\sum_{n=1}^{m} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{n=1}^{m} (x_i - \bar{x})^2} \sqrt{\sum_{n=1}^{m} (y_i - \bar{y})^2}}
\end{align*}
\]  

-1 implies negative correlation

0 implies no correlation

1 implies positive correlation
Sampling Method: A Visualization

- **Cube** - $M(s, c)$
- **Square** - $M(s, c, d \pm n)$
- **Curve** - $M(s, c, d)$
Goals

Main Question
Will we be able to obtain more accurate and consistent results if we do not allow for the $\pm n$ error on $d$?

- Determine if the $\pm n$ error allowed on $d$ made a difference in our correlation results or not.
- Create code that would perform the raster marginals model with coupling terms but without the error on $d$.
- Create a program that would generate multiple sample matrices, both with and without the $\pm n$ on $d$ and output their corresponding correlations.
Modified Code & New Program

1. Inputs: raster file, number of columns, number of sample matrices from Dr. Okun’s original code, and number of sample matrices from our modified code.

2. Generate a new raster file of a specified size.

3. Extract the three parameters and run Dr. Okun’s code and our modified version of his code the specified number of times.

4. Calculate the correlation coefficient matrix.

5. Calculate the estimated standard deviations for both sample correlation values and compute the difference.

6. Plot the correlations.
Explanation of Graphs

- Each run of the program produces \( n \) graphs, where each graph represents the correlation between neuron \( i \) and all other neurons.
- The \( x \)-axis represents the neurons and the \( y \)-axis represents the correlation.
- Here we use 100 samples each from Dr. Okun’s code and our modified code.
  - Red dots - Dr. Okun’s code
  - Blue dots - Modified code
  - Green dots - new raster file
Correlation between Neuron 5 and Other Neurons

- Correlation
- Neuron N

The diagram shows the correlation between Neuron 5 and other neurons, with a scatter plot indicating the correlation values for each neuron.
10 × 300 Raster Example

Correlation between Neuron 5 and Other Neurons

Legend:
- pink: d ± n
- light blue: d
- green: Raster

Correlation vs Neuron N graph showing data points scattered across the range.
10 × 3000 Raster Example
10 × 10000 Raster Example
10 × 170000 Raster Example

Correlation between Neuron 7 and Other Neurons

Correlation

Neuron N
A Closer Look

Correlation between Neuron 7 and Other Neurons
As the number of columns increases, the tolerance on $d$ matters less.

As the number of columns increases, the more similar the standard deviations of both samples become.

Losing the original permutation of $c$ seems to have an effect on correlations.

Important to note: these results and claims can only be applied to similar rasters.
Future Directions

- Test our claims and results on various rasters that we did not have access to.
- Determine if the provided three parameters are actually enough to determine pairwise correlations between pairs of neurons.
- Find bounds on the solution spaces for matrices with prescribed row sum, column sum, and inner product constraints.
Thank you

I would like to first and foremost thank our mentor, Dr. Anne Shiu. This research experience would not have been possible if not for her mentorship and guidance. I would also like to thank Kaitlyn Phillipson, Ola Sobieska, and Robert Williams for their assistance on this project, as well as my partner Adriana Morales. Finally, I would like to thank Dr. Michael Okun for generously answering our questions and providing data for this research.
References

