An Application of Compressive Sensing to Image and Video Compression

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Compressive Sensing is a relatively young area of Signal Processing that deals with compressing and reconstructing linearly-modeled signals.
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Compressive Sensing works in a 'naive' manner, requiring no prior knowledge of the signal and instead relying on the structure that are often found in linearly-modeled signals.
Suppose $\mathbf{x} \in \mathbb{R}^n$ is our signal that we are interested in compressing. We perform the compression by multiplying $\mathbf{x}$ by $\Phi$, an $m \times n$-matrix, where $m \ll n$.

$$\mathbf{y} = \Phi \mathbf{x}$$  

Thus $\mathbf{y}$ represents our compressed signal. By imposing conditions on $\mathbf{x}$ and $\Phi$, we can recover our signal.
The Restricted Isometry Property

A signal can be recovered if there exists a $\delta_K \in (0, 1)$, where the $\Phi$ matrix satisfies

$$(1 - \delta_K) \|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta_K) \|x\|_2^2.$$  \hspace{1cm} (2)

where $x \in \Sigma_K = \{x : \|x\|_0 \leq K\}$, $\| \cdot \|$ denoting the sparsity of the vector, the number of nonzero entries. This property is known as the Restricted Isometry Property (RIP).
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- Construct $Phi$ by choosing the entries from a Normal distribution with zero mean and a standard deviation of $m^{-1}$.
- Construct $Phi$ by randomly choosing $m$ distinct rows of a wavelet matrix.
[6] Let $\Phi$ be an $m \times n$-matrix that satisfies the RIP of order $2K$ with constant $\delta \in (0, \frac{1}{2}]$. Then

$$m \geq C \log \left( \frac{N}{K} \right)$$

(3)

where $C = (2 \log(\sqrt{24} + 1))^{-1}$. 

Theorem
[6] If

\[ K < \frac{1}{2} \left( 1 + \frac{1}{\mu(\Phi)} \right) \]

where \( \mu(\Phi) = \max_{1 \leq i < j \leq n} \frac{|\langle \phi_i, \phi_j \rangle|}{\|\phi_i\|_2 \|\phi_j\|_2} \) (4)

then for each measurement vector \( y \in \mathbb{R}^m \) there exists at most one signal \( x \in \Sigma_K \) such that \( y = \Phi x \).
To recover our original signal, we solve the convex optimization problem

$$\min_x \| y - \Phi x \|_2 + \| x \|_1$$

(5)

Algorithms such as linear programming and gradient descent can be used.

The algorithm we use is called the *Multihypothesis Block-based Compressive Sensing*.
Donoho-Tanner Phase Transition [7]

stepwise with FDR threshold, $z \sim N(0, 16)$,
normalized $L_2$ error, $p = 200$

$\rho = k/n$

$\delta = n/p$
A satellite takes a picture while in flight.
The image is then separated into a red, green, and blue channels.
Each Channel is then taken and multiplied by a different $\Phi$ Matrix.
The picture is then received on Earth.

Each individual channel is reconstructed by parallel computing using a cluster of computers.

After each channel is reconstructed the channels are combined back into one picture.
Reconstructed Images

320x320
Reconstructed Images Cont.

600x600
Reconstructed Images Cont.

1024 x 1024
Reconstructed Images Cont.

1024 x 1024
Reconstructed Images Cont.

1920 x 1920
Design of our Algorithm: Video

- The video is taken and broken up into frames.
- Each frame is treated as image and compressed then reconstructed using the same procedure as in the previous two slides.
- The main difference is that in this code, after the 1st frame, the previous frame is used as an initial guess.
E. Candés, J. Romberg, and T. Tao,
*Stable signal recovery from incomplete and inaccurate information*,

E. Candés, T. Tao,
*The Dantzig selector. Statistical estimation when p is much larger than n*,

E. Candés, T. Tao,
*Near optimal signal recovery from random projections: Universal encoding strategies*,
E. Candés, J. Romberg, and T. Tao,
*Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information*,

D. Donoho,
*Compressed Sensing*,

Richard Baraniuk, Mark A. Davenport, Marco F. Duarte, Chinmay Hegde, Jason Laska, Mona Sheikh, Wotao Yin,
*An Introduction to Compressive Sensing*,
Connexions, Rice University, Houston, Texas, Online: <http://cnx.org/content/col11133/1.5/>.
David Donoho, Jared Tanner,

*Observed Universality of Phase Transitions in High-Dimensional Geometry, with Implications for Modern Data Analysis and Signal Processing.*