

## WEEK 2 REVIEW

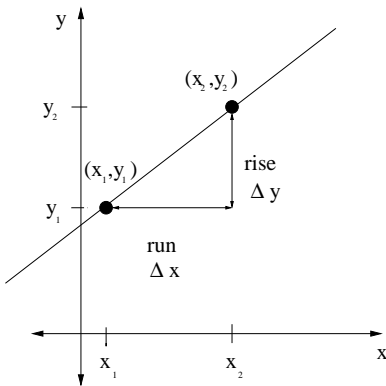
- Straight Lines (1.2)
- Linear Models (1.3)
- Intersection Points (1.4)
- Least Squares (1.5)

## STRAIGHT LINES

### SLOPE

A VERTICAL line has NO SLOPE. All other lines have a slope given by

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Example - what is the slope of the line passing through the points  $(-2, 4)$  and  $(0, -4)$ ?

Answer - Let one pair of points be  $(x_1, y_1)$  and the other  $(x_2, y_2)$  and then plug into the formula,

$$m = \frac{4 - (-4)}{-2 - 0} = \frac{8}{-2} = -4$$

if we assigned our points the other way we would have

$$m = \frac{-4 - 4}{0 - (-2)} = \frac{-8}{2} = -4$$

so you just need to be consistent.

## EQUATIONS OF LINES

The formula for the slope of a line can be rearranged to give us the equation for a line.

$$y - y_1 = m(x - x_1)$$

this is called the POINT-SLOPE form. If you know a point,  $(x_1, y_1)$  that lies on the line and you know the slope,  $m$ , of the line, then you can find the equation of the line.

Example - What is the equation of the line passing through the points  $(-2,4)$  and  $(0, -4)$ ?

Answer - We have two points rather than the slope and one point. So we first need to calculate the slope. This was done in the last example and so we know the slope is  $m = -4$ . Choose our point to be  $(-2, 4)$  and plug in,

$$y - 4 = -4(x - (-2)) = -4(x + 2) = -4x - 8$$

$$y = -4x - 8 + 4 = -4x - 4$$

We can also choose  $(0, -4)$  as our point and find

$$y - (-4) = -4(x - 0) \rightarrow y + 4 = -4x \rightarrow y = -4x - 4$$

Same equation both ways! When we simplify our point-slope form we are writing the line in the slope-intercept form,

$$y = mx + b$$

Again,  $m$  is the slope and now  $b$  is the  $y$ -intercept. The  $y$ -intercept is the place where the line crosses the  $y$ -axis. The  $y$ -axis is the line  $x = 0$  and we see if we plug  $x = 0$  into our equation we find  $y = b$ .

We can also find the  $x$ -intercept. This is where the line crosses the  $x$ -axis. The  $x$ -axis is the line  $y = 0$ . For the line  $y = -4x - 4$  we find the  $y$ -intercept as the place where  $x = 0$ ,

$$y = -4(0) - 4 = -4, \text{ so } y\text{-intercept is } (0,-4)$$

find the  $x$ -intercept where  $y = 0$ ,

$$0 = -4x - 4 \rightarrow 4 = -4x \rightarrow x = -1, \text{ so } x\text{-intercept is } (-1,0).$$

It is very important to be able to find the intercepts of any graph. When you are deciding the window for your calculator screen or the scale on your sketch be sure that the intercepts can be seen.

Finally, the equation of a line can be expressed in GENERAL FORM,

$$Ax + By = C$$

This form is often the form you are given in a problem. It is a very useful for for finding the intercepts - just take turns letting  $x$  or  $y$  be zero. However, if you want to put it into your calculator you must write it in slope-intercept form.

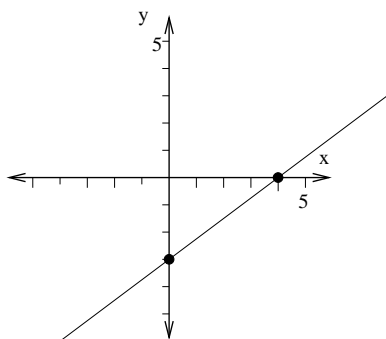
Example - Graph the line  $3x - 4y = 12$  on paper and on the calculator.

Answer - I would start by finding the intercepts,

Let  $x = 0$  and the equations looks like  $3(0) - 4y = 12 \rightarrow -4y = 12 \rightarrow y = 12 / -4 = -3$ . So the  $y$ -intercept is at  $(0, -3)$ .

Let  $y = 0$  and the equation looks like  $3x - 4(0) = 12 \rightarrow 3x = 12 \rightarrow x = 4$ . So the  $x$ -intercept is  $(4, 0)$ .

I want to choose a scale on my paper that will let me graph the points  $(0, -3)$  and  $(4, 0)$ . How about letting  $x$  and  $y$  go from  $-5$  to  $5$ . Draw the axes. LABEL THE AXES. Put in the tick marks. LABEL THE TICK MARKS. Mark the intercepts. Draw a line thru the two points.



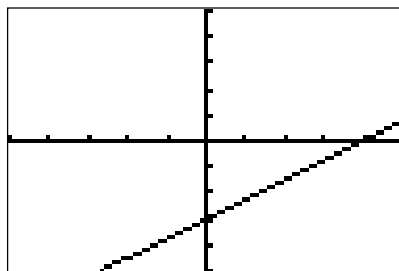
Now to graph on the calculator I need to rewrite the equation in the slope-intercept form (looks like  $y = \dots$ ),

$$3x - 4y = 12 \rightarrow -4y = -3x + 12 \rightarrow y = (-3x + 12)/(-4) = (3/4)x - 3$$

Put this into the calculator. Now set the window. I know where the intercepts are and so I will choose window values so that I can see these intercepts. The XSCL and YSCL show how often there are tick marks, when they are equal to 1, the tick marks appear every one unit.

```
Plot1 Plot2 Plot3
Y1=(3/4)X-3
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-5
Ymax=5
Yscl=1
Xres=1
```



We find that two lines are parallel if they have the same slope and two line are perpendicular if the product of their slopes is  $-1$

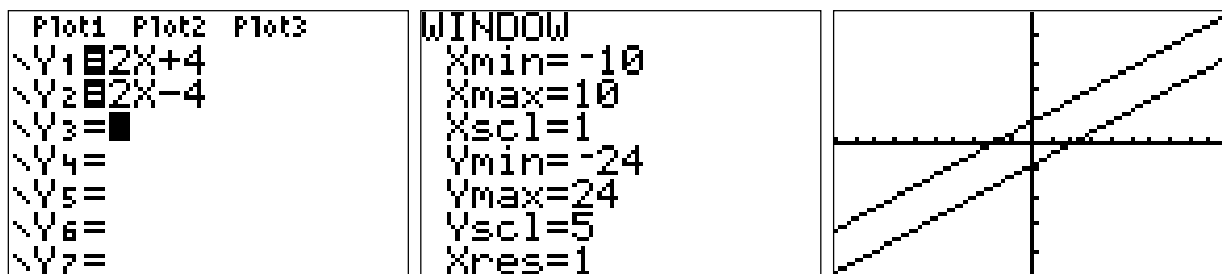
Example: Given the line  $y = 2x + 4$ ,

- (a) find a line parallel to this line that passes through the point (4,4)
- (b) find a line perpendicular to this line that passes through the point (4,4)

Answer: Start by find the slope of our given line. It is in slope-intercept form, so I can read that  $m = 2$ . Now I need the line that has a slope of 2 that passes through the point (4,4):

$$y - 4 = 2(x - 4) = 2x - 8 \rightarrow y = 2x - 8 + 4 = 2x - 4$$

We can (sort of) check this on the calculator by graphing both lines and looking at them.



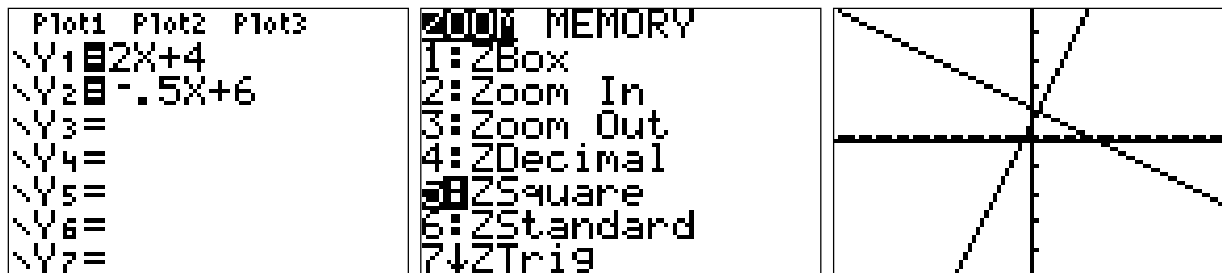
Now we want to have a perpendicular line, so we need to use

$$m_1 \cdot m_2 = -1 \rightarrow 2 \cdot m_2 = -1 \rightarrow m_2 = -1/2$$

So the slope of our new line is  $-1/2$  and it passes through the point (4,4),

$$y - 4 = (-1/2)(x - 4) = (-1/2)x + 2 \rightarrow y = (-1/2)x + 2 + 4 = (-1/2)x + 6$$

Check on the calculator to see if it looks perpendicular... (use zoomsquare or ratio of pixels is wrong)



REMEMBER - your calculator is a TOOL. it will only do what you tell it! you should have some idea how your graph should look! Mistakes with parentheses have caused much grief in the past - don't let it happen to you. (Look at  $3/4x-3$ ). You should be careful about the graphs you turn in for grading - be certain things are properly labeled and the domain is correct (next example). You do not have to graph a line on your calculator - it is often easier to do it by hand.

## APPLICATIONS

Many “word problems” use lines as models. We will discuss this more in the next sections, but one point to look out for is that we want to only graph  $x$  and  $y$  values for things that make sense.

Example - For wages less than the maximum taxable wage base, Social Security contributions by employees are 7.65% of the employee’s wages.

a) Let  $x$  be the amount of the employee’s wages and  $y$  the amount of the social security contribution. Find an equation that expresses the relationship between  $x$  and  $y$  for employees earning less than the maximum (\$68,400 in 1998)

b) Graph this equation and find the social security contribution for an employee earning \$35,000 in wages in a year.

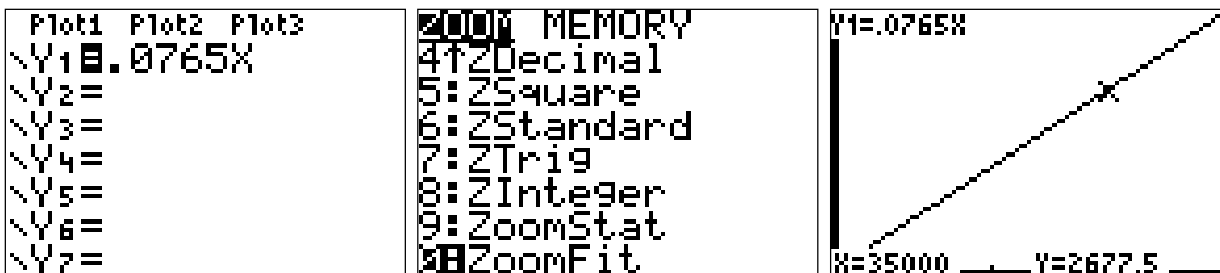
Answer - (a) For each dollar earned, you pay 7.65% or \$.0765. So we can find two points, (0,0) and (100, 7.65). Find the slope and use the point-slope equation to find

$$y = .0765x$$

(b) Before graphing, we need to think of the domain! Since  $x$  is wages, the least you can earn is 0. We are told the maximum for this model is 68,400 and so our domain is  $0 \leq x \leq 68,400$ . We will use this for our limits on  $x$ . An easy way to find the limits on  $y$  is to use the ZOOMFIT feature. To find the social security paid when the wages are 35,000 we can plug into our equation,

$$y = .0765x = .0765 \cdot 35,000 = 2677.5 \text{ or the employee paid } \$2677.50.$$

OR, you can use the value function on your calculator since you already put the equation in.



## LINEAR MODELS

You must be familiar with the following six models:

- Depreciation - the value ( $V$ ) of an item decreases linearly with time. The item has a certain initial value and the value decreases by the same amount each time period.
- Cost - in a linear cost model the cost to make  $x$  items is  $C(x) = cx + F$ .  $F$  is the *fixed costs*. These are the costs you have even if you make no items.  $c$  is the cost to make each unit, called the *variable cost*. If you are given two points with the total costs, you must find  $F$  and  $c$  using the point-slope form of a line. Read the information you are given carefully!
- Revenue - in a linear revenue model if we have  $x$  items are sold for  $s$  dollars each, then the company brings in  $sx$  dollars. This is the revenue,  $R(x) = sx$ .
- Profit - The difference between the money in (revenue) and the money spent (costs) is the profit. Profit can be positive or negative.

$$P(x) = R(x) - C(x)$$

- Supply - If a company is supplying an item, it will supply more of this item when they can get more money for it. We have  $x$  for the number of items supplied and the price,  $p = S(x)$ .
- Demand - If an item costs less money, the consumer will buy more of it. We have  $x$  for the number of items bought by the consumer and the price  $p = D(x)$ .

## STRAIGHT-LINE DEPRECIATION

This models how an item loses value over time,  $V = mx + b$ . We will find  $m$  is negative for those items whose value decreases over time.

Example: A car is purchased for \$18,000 and is kept for 7 years. At the end of 7 years the car is sold for \$4000. Find an equation that models the decrease in the value of the car over time. What is the car worth after 3 years?

Answer: We again have two data points. We will let  $x$  be the number of years and  $y$  be the value of the car. Our points are then  $(0, 18000)$  and  $(7, 4000)$ . Find the slope,

$$m = \frac{18000 - 4000}{0 - 7} = \frac{14000}{-7} = -2000$$

The slope tells us that the car decreases in value by \$2000 each year. Find the equation of the line,

$y - 18000 = -2000(x - 0) \rightarrow y = -2000x + 18000$  or since we are to find the value, write it as

$$V(x) = -2000x + 18000$$

after 3 years the value will be  $X = V(3) = -2000(3) + 18,000 = 12,000$ , so the car is worth \$12,000.



## COST, REVENUE and PROFIT

Example - Suppose a company manufactures baseball caps. In a day they can produce 100 caps at a total cost of \$600. If no caps are produced their costs are \$200 per day. The caps sell for \$8 each. Find the profit equation.

Answer - start by finding the cost equation. We are given the total cost to produce 100 caps:

$$C(x) = mx + b \rightarrow C(100) = 600 = m(100) + b$$

We are also given the cost to produce zero caps:

$$C(x) = mx + b \rightarrow C(0) = 200 = m(0) + b = b \rightarrow b = 200$$

So our fixed costs are \$200 and we can now find the cost to make each cap:

$$600 = m(100) + 200 \rightarrow m = 4. \text{ So the variable cost (cost each) is } \$4.$$

Our final cost equation then is

$$C(x) = 4x + 200$$

It is wise to check that if you make zero caps, it costs \$200 and if you make 100 caps it costs \$600.

The revenue equation is the price each is sold for, here  $p = 8$ , times how many are sold,  $x$ :

$$R(x) = 8x$$

Finally, the profit is the difference between the money in (revenue) and the money spent (cost):

$$P(x) = R(x) - C(x) = [8x] - [4x + 200] = 8x - 4x - 200 = 4x - 200$$

## INTERSECTION OF TWO LINES

If two lines have different slopes they will intersect somewhere. The point where the two lines cross is the place where both equations are true at the same time. Therefore this intersection point is called the solution to the two equations. The intersections point can be found using algebra or using the calculator. You should be familiar with both. There are many applications of the intersection of two lines. We will look at two of them:

Break-even point: This is where the cost to produce items is the same as the revenue brought in from selling the items.  
 $R(x) = C(x)$ .

Example: A firm producing dog food finds that the total cost to produce  $x$  units of food is given by

$$C(x) = 10x + 200.$$

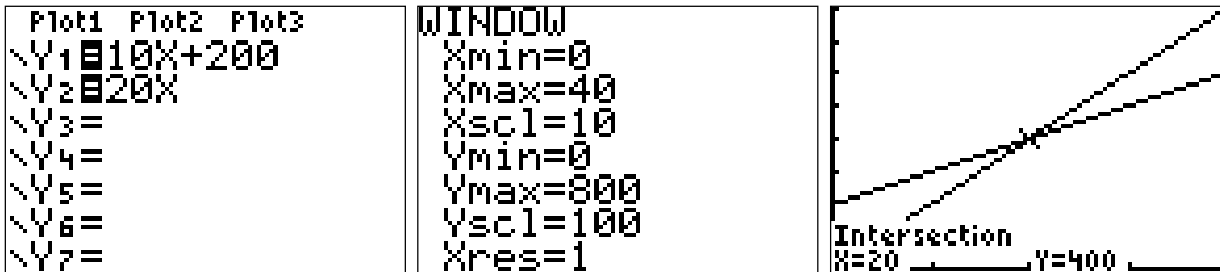
The food will be sold for \$20 per unit. How many units must be sold to break-even?

$R(x) = 20x = C(x) = 10x + 200$ . subtract  $10x$  from both sides,

$10x = 200$  divide both sides by 10

$x = 20$  or 20 units must be sold to break-even.

On the calculator, graph both equations and use the intersection option under the CALC menu.



## DEMAND

For most products sold, how many people will buy depends on the price. Usually if the price is lower for an item, people will buy more of it. We can model this behavior with the demand equation,

$$D(x) = p = mx + b$$

where  $p$  is the price and  $x$  is the number of items purchased by the consumer. This line slopes downward as more items are sold when the price is lower.

example: A company finds that it can sell 100 disc players per day if the price is \$50 and sell 70 disc players per day if the price is \$80. (notice the higher price sells fewer). Find the demand equation for disc players.

answer: we have two data points and we need to find the equation of the line that passes through them. We must be sure they are written as ordered pairs  $(x, p)$ :  $(100, 50)$  and  $(70, 80)$  next find the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{50 - 80}{100 - 70} = \frac{-30}{30} = -1$$

Now use the point-slope form to get the line:

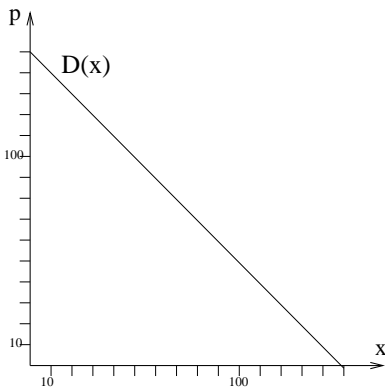
$$y - y_1 = m(x - x_1) \rightarrow y - 50 = -1(x - 100) \rightarrow y = -x + 100 + 50 = -x + 150$$

Now remember, this is the demand equation modeling the change in the price with respect to the number of items so I need to write this as

$$D(x) = p = -x + 150$$

Again it is wise to check that if 100 disc players were sold that the price was \$50 and if 70 disc players were sold that the price was \$80.

Graph the equation. We can do it on the calculator, but by hand is about as easy ....



## SUPPLY

When a manufacturer is selling an item, if the price is higher they will supply more of the item than if the price was lower. The model for the number of items supplied and the price paid for them is called the supply equation,

$$S(x) = p = mx + b$$

$p$  is the price and  $x$  is the number of items supplied by the manufacturer. This line generally slopes upward as if the supplier can get a higher price they will supply more of that item.

example: a company manufactures disc players. They are willing to supply 30 disc players per day when the price they get is \$30 each, but they will supply 60 when the price each is \$45. Find the supply equation.

answer: Find our two data points. again they are  $(x, p)$  pairs: (30, 30) and (60, 45). find the slope,

$$m = \frac{45 - 30}{60 - 30} = \frac{15}{30} = .5$$

Now find the line,

$$y - 30 = .5(x - 30) \rightarrow y = .5x - 15 + 30 = .5x + 15$$

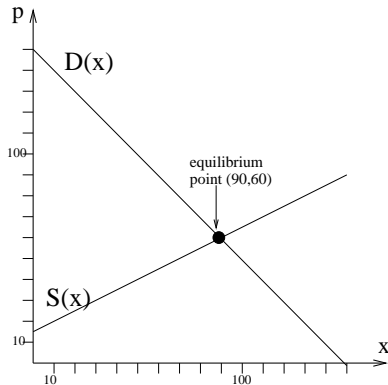
So we would write the supply equation as

$$S(x) = p = .5x + 15$$

Once again we check that our initial data works....

## EQUILIBRIUM POINT

This is the quantity,  $x_0$ , and price,  $p_0$ , that the consumer and producer agree upon. That is, the consumer is willing to buy  $x_0$  items at a price  $p_0$  and the supplier is willing to supply that many at that price. This occurs where the supply and demand equations intersect.



Graph this equation on the same screen/page as the demand equation. [window  $x=0$  to  $120$  and  $y=0$  to  $200$ ] We see that they intersect. The point of intersection is the equilibrium point. This is the quantity and price that both the supplier and consumer can agree on. The  $x$ -value is the equilibrium quantity and the  $p$ -value is the equilibrium price. We can find the quantity using algebra or the calculator.

At the equilibrium point the two prices are the same, so I can set them equal:

$$p = p \rightarrow S(x) = D(x) \rightarrow .5x + 15 = -x + 150 \rightarrow 1.5x = 135 \rightarrow x = 90$$

Now find the price using either equation,

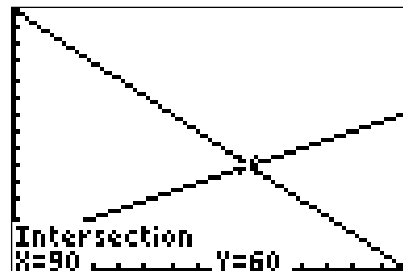
$$p = D(90) = -90 + 150 = 60$$

$$p = S(90) = .5(90) + 15 = 60.$$

So the equilibrium point is  $(90,60)$ .

```
Plot1 Plot2 Plot3
Y1=.5X+15
Y2=-X+150
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
WINDOW
Xmin=0
Xmax=150
Xscl=10
Ymin=0
Ymax=150
Yscl=10
Xres=
```



## LEAST SQUARES

When you have more than two data points, you want to find a line that comes as close as possible to all the data points. Your calculator automates this for you. You need only put the data in and the least-squares line (or regression equation) will be calculated for you. You can then use the least-squares line as a model to make predictions.

Example: We are given the following data about square footage and air conditioner BTU's:

ft <sup>2</sup>	150	215	280	310	450
BTUs	5000	6000	7000	7500	9500

Graph the data and find the least-squares line. How many BTUs would you need for a room that is 400 ft<sup>2</sup>? If the air-conditioner is 5500 BTUs, how large would you estimate the room to be?

<pre> EDIT  [0] [1] [2] [3] [4] [5] [6] [7] [8] [9] CALC TESTS 1: Edit... 2: SortA( 3: SortD( 4: ClrList 5: SetUpEditor         </pre>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th>L1</th> <th>L2</th> <th>L3</th> <th>2</th> </tr> <tr> <td>150.00</td> <td>5000.0</td> <td>-----</td> <td></td> </tr> <tr> <td>215.00</td> <td>6000.0</td> <td></td> <td></td> </tr> <tr> <td>280.00</td> <td>7000.0</td> <td></td> <td></td> </tr> <tr> <td>310.00</td> <td>7500.0</td> <td></td> <td></td> </tr> <tr> <td>450.00</td> <td>9500.0</td> <td></td> <td></td> </tr> <tr> <td>-----</td> <td>-----</td> <td></td> <td></td> </tr> <tr> <td colspan="4">L2(6) =</td> </tr> </table>	L1	L2	L3	2	150.00	5000.0	-----		215.00	6000.0			280.00	7000.0			310.00	7500.0			450.00	9500.0			-----	-----			L2(6) =				<pre> EDIT  [0] [1] [2] [3] [4] [5] [6] [7] [8] [9] TESTS 1: 1-Var Stats 2: 2-Var Stats 3: Med-Med 4: LinReg(ax+b) 5: QuadReg 6: CubicReg 7: QuartReg         </pre>
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<pre> LinReg(ax+b) L1, L2         </pre>	<pre> LinReg y=ax+b a=15.0236 b=2778.3778 r^2=.9994 r=.9997         </pre>
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If your calculator screen doesn't show  $r$  and  $r^2$  you need to go to the CATALOG button (above the 0) and choose DiagnosticsOn and enter.

<pre> CATALOG Degree DelVar DependAsk DependAuto det( DiagnosticOff DiagnosticOn         </pre>	<pre> DiagnosticOn Done         </pre>
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If you want to see the least squares line along with the data and use it to answer the questions it is easiest to run the LEASTSQR program:

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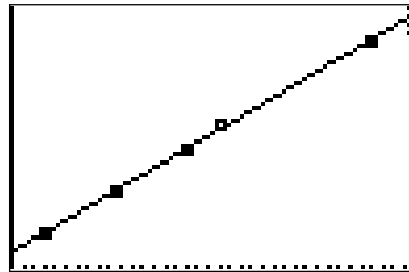
EXEC EDIT NEW
1:BIN
2:DRAWNORM
3:LEASTSQ
4:NORMAREA
5:ROWOPS
6:SMPLX

```

```

PRGMLEASTSQ

```



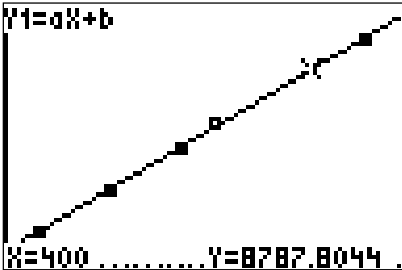
To get out of the graph screen, push the ON button,

```

PRGMLEASTSQ
Y IS AX+B WHERE
A,B, AND R ARE
      15.0236
    2778.37700
      .9997
      Done

```

To now use the line, you can hit GRAPH again and find the values needed. To find the X for a certain Y, you need to graph the Y value and find where it intersects the line:



```

Plot1 Plot2 Plot3
Y1=aX+b
Y2=5500
Y3=
Y4=
Y5=
Y6=
Y7=

```

