Homework 10

Math 300 (section 901), Fall 2021

This homework is due on Wed., Nov. 3. (Turn in your answers to questions 1–7.)
You may cite results from class, as appropriate.

0. *(This problem is NOT to be turned in.)*

   (a) Read Sections 9.1–9.3
   (b) Section 6.3 #6.35
   (c) Section 9.1 #9.1, 9.5
   (d) Section 9.2 #9.11, 9.17

1. Recall that a *prime* number is an integer $p \geq 2$ such that the only positive integers that divide p are 1 and p. Prove that there are infinitely many prime numbers. (*Hint:* Proceed by contradiction: Assume that there are finitely many primes, $p_1, p_2, \ldots, p_m$. Consider the integer $n = 1 + p_1 p_2 \ldots p_m$.)

2. The goal of this problem is to prove (by strong induction) the following: *Every integer $n \geq 2$ is a prime or is a product of primes.*

   (a) State the **base case** for a proof (of the statement) by strong induction (on n).
   (b) Prove the base case.
   (c) State the **inductive hypothesis.**
   (d) State the *goal* of the **inductive step**.
   (e) Complete the inductive step,

3. *(No proofs necessary for this problem.)* Let $A = \{a, b, c, d\}$.

   (a) Give an example of a relation on A that is reflexive and symmetric, but not transitive.
   (b) Give an example of a relation on A that is reflexive and transitive, but not symmetric.
   (c) Give an example of a relation on A that is symmetric and transitive, but not reflexive.

4. Consider the relation R on $\mathbb{R}$ defined by aRb if $ab > 0$.

   (a) Is R reflexive? (Prove your answer.)
   (b) Is R symmetric? (Prove your answer.)
   (c) Is R transitive? (Prove your answer.)

5. Consider the relation R on $\mathbb{R}$ defined by aRb if $a|b$ or $b|a$.

   (a) Is R reflexive? (Prove your answer.)
   (b) Is R symmetric? (Prove your answer.)
   (c) Is R transitive? (Prove your answer.)

6. Section 9.1 #9.2, 9.6, 9.8

7. Section 9.2 #9.12, 9.16 (and explain your answer to this problem)

Writing Assignment 5

Math 300, Fall 2021

This homework is due on **Wednesday, Nov. 3**.

This week, you will critique another student's draft. Staple your comments to the draft, and make sure both your name and the author's name appear on the front. Do not staple this to the rest of your homework.

(a) Read through the draft. Mark each spot you got confused.

(b) What questions do you have for the authors?

(c) What aspects of the draft did you like?

(d) What could the author do to improve this draft?

Writing Assignment 6

This homework is due on **Wednesday, Nov. 10**.

- Extend your draft to include four sections of your final paper:

    1. the introduction (what will your paper be about?),
    2. the mathematical background (define and/or explain all unfamiliar terms), and
    3. *two* sections developing main ideas from Writing Assignment 3.

- *Edit your draft in response to all comments (to be given next week).*

- The required length is at least four pages.

- *Please print 1 copy of your draft.*

- If you do **not** turn in this draft, you will receive a 5% penalty on the final paper.