

Homework 13

Math 300 (section 901), Fall 2021

This homework is due on MONDAY, Nov. 29¹. (Turn in your answers to questions 1–8.) You may cite results from class, as appropriate.

0. (*This problem is NOT to be turned in.*)
- (a) Read Sections 10.3–10.5
 - (b) If A and B are nonempty sets, does it follow that there exists a function $f : A \rightarrow B$?
 - (c) Is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = -4x + 3$ bijective? If so, find the inverse.
 - (d) Is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x + |x|$, bijective? (Prove your answer.)
 - (e) Section 10.3 #10.32
 - (f) Section 10.4 #10.41, 10.46
1. Prove or disprove: *For a function $f : A \rightarrow B$, if $X \subseteq Y \subseteq B$, then $f^{-1}(X) \subseteq f^{-1}(Y)$.*
2. Prove the following (which we stated in class but did not prove): *If $f : A \rightarrow B$ is a function, with $|A| = |B| < \infty$, then f is one-to-one if and only if f is onto.* (Hint: Use the lemma from class that states that, when A is finite, $|\text{range}(f)| \leq |A|$, with equality if and only if f is one-to-one.)
3. Consider the following function:

$$f : \mathbb{R} - \left\{ \frac{3}{5} \right\} \rightarrow \mathbb{R} - \left\{ \frac{2}{5} \right\}$$
$$x \mapsto \frac{2x + 1}{5x - 3}.$$

Is it bijective? Prove your answer.

4. Is the following function $f : \mathbb{R} \rightarrow \mathbb{R}$ bijective? If so, find the inverse function:

$$f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0. \end{cases}$$

5. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove or disprove the following:
- (a) If $g \circ f$ is surjective, then f is surjective.
 - (b) If $g \circ f$ is surjective, then g is surjective.
 - (c) If $g \circ f$ is injective, then f is injective.
 - (d) If $g \circ f$ is injective, then g is injective.
6. For each of the statements in #5, state the converse, and then prove or disprove it.
7. Section 10.3 #10.33
8. Section 10.4 #10.37, 10.38, 10.39, 10.40

¹As a reminder, the next draft of your report is also due on Nov. 29.