Homework 14

Math 300 (section 901), Fall 2021

This homework is due on Wed., Dec. 8 (the last day of class). (Turn in your answers to questions 1–8.) You may cite results from class, as appropriate.

0. *(This problem is NOT to be turned in.)*
   (a) Read Sections 11.1–11.5 and 12.1–12.3
   (b) Section 10.5 #10.50, 10.52, 10.53,
   (c) Section 11.2 #11.3, 11.5
   (d) Section 11.3 #11.25
1. Prove the following: *If $f : A \to B$ and $g : B \to C$ are bijective functions, then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.*
2. Assume that a, b, c, d are real numbers with $a \neq 0$ and $c \neq 0$. Let $f(x) = ax + b$ and $g(x) = cx + d$ be functions (with both domain and codomain equal to $\mathbb{R}$). Is the function $h := g \circ (f^{-1})$ bijective? If so, find the inverse function of h (and prove that it is the inverse).
3. Let $f : A \to B$ be a function. Let Id_A and Id_B denote the identity functions on A and B, respectively. Prove or disprove the following:
   (a) If there exists a function $h : B \to A$ such that $h \circ f = Id_A$, then f is surjective (onto).
   (b) If there exists a function $h : B \to A$ such that $h \circ f = Id_A$, then f is injective (one-to-one).
   (c) If there exists a function $h : B \to A$ such that $f \circ h = Id_B$, then f is surjective.
   (d) If there exists a function $h : B \to A$ such that $f \circ h = Id_B$, then f is injective.
4. Is $\mathbb{Z} \times \{1, 2\}$ countable? Prove your answer.
5. Is $\mathbb{Z} \times \mathbb{Q}$ countable? Prove your answer.
6. Prove or disprove: *If A and B are nonempty sets, then $|A| \leq |A \times B|$.*
7. Use the Schröder-Bernstein Theorem to prove that the intervals $[0, 1]$ and $[1, 5)$ have the same cardinality.
8. (a) Section 10.5 #10.56, 10.58
   (b) Section 11.2 #11.12
   (c) Section 11.3 #11.20, 11.24
   (d) Section 11.4 #11.28
   (e) Section 12.1 #12.2, 12.14
   (f) Section 12.2 #12.15(a–b), 12.18
   (g) Section 12.3 #12.34