Homework 4

Math 300 (section 901), Fall 2021

This homework is due on Wed., Sept. 22. (Turn in your answers to questions 1–11.)
You may cite results from class, as appropriate.

0. *(This problem is NOT to be turned in.)*

    (a) Read Sections 2.5–2.6, 2.8–2.10, and 3.2

    (b) Skim Section 2.11.

    (c) Section 2.3 #2.15, 2.16

    (d) Section 2.4 #2.20, 2.28

    (e) Section 2.5 #2.30

    (f) Section 2.6 #2.42

    (g) Section 2.8 #2.54

    (h) Section 2.9 #2.64, 2.66

    (i) Section 2.10 #2.70, 2.76

    (j) Section 3.2 #3.10

    (k) Use a truth table to verify the distributive laws for statements (Theorem 2.22.3).

    (l) Negate the following:

        (i) "The real number x is positive."

        (ii) "The function f is undefined at $x = 1$."

1. Read Section 2.7 and summarize its content in 1–2 sentences.

2. Which example in Section 2.10 do you find to be the most interesting and/or confusing? Explain briefly.

3. Let P and Q be statements.

    (a) If $(\sim P) \vee Q$ is true, does this imply that $P \vee Q$ is false? Explain.

    (b) If $(\sim P) \wedge (\sim Q)$ is true, does this imply that $P \vee Q$ is false? Explain.

4. Determine all values of S in the domain $\mathcal{P}(\{a, b, c\})$ for which the following is a true statement:
   "The set S contains the element b $\iff$ The set $\{a, b, c\} - S$ contains the element c."

5. (a) What is the *converse* of an implication $P \Rightarrow Q$? (See pg. 53.)

    (b) Are $P \Rightarrow Q$ and its converse, logically equivalent? (Use a truth table.)

    (c) The *contrapositive* of $P \Rightarrow Q$ is $(\sim Q) \Rightarrow (\sim P)$. Are $P \Rightarrow Q$ and its contrapositive, logically equivalent?

6. Negate the following:

    (a) Every integer is nonzero.

    (b) If S is a finite set, then $S \cup \{a, b, c\}$ is a finite set.

7. Assume that S and T are sets, and $P(x)$ and $Q(y)$ are open sentences over the domains S and T, respectively.) Negate the following:

   (a) $\forall x \in S, \ \sim P(x)$

   (b) $\exists x \in S, \ \forall y \in T, \ (P(x) \vee Q(y))$.

8. Determine whether each statement is true or false. Explain your answer.

   (a) For every real number x, the equality $x^2 - 6x + 9 = 0$ holds if and only if $x = 3$.

   (b) For every real number x, the equality $x^2 - 2x - 3 = 0$ holds if and only if $x = 3$.

   (c) For every real number x, the equality $x^2 + 3 = 0$ holds if and only if $x = 3$.

   (d) For every real number x, if the equality $x^2 + 3 = 0$ holds, then $x = 3$.

   (e) For every real number x, there exists a real number y such that $x - y = 8$.

9. Prove the following: *If n is an even integer, then $2n - 1$ is odd.*

10. Suggest two problems for the first midterm exam (which is on Friday, October 8):

    • one from the Chapter 2 Supplementary Exercises, and

    • another one on any topic from Chapter 2.

11. (a) Section 2.3 #2.18

    (b) Section 2.4 #2.24, 2.26

    (c) Section 2.5 #2.34(a–d) – Also, indicate whether each is true or false.

    (d) Section 2.6 #2.36 – Also, indicate whether each is true or false.

    (e) Section 2.9 #2.62(b–d)

    (f) Section 2.10 #2.68

Writing Assignment 3

Math 300 (section 901), Fall 2021

This homework is due on Wed., Oct. 6 (so, you have two weeks to complete this). *Complete this part on a separate piece of paper, not the same paper for Homework.*

1. List the sources you plan to use for your term paper (websites, articles, reference books, etc.)

2. Write one paragraph describing what you expect to be the main message[1] of your paper.

3. List three key ideas that you expect to develop in your paper.

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[1]Here, you need to state the message – the main take-away for your reader – not the actual mathematical content of the paper.