Homework 6

Math 300 (section 901), Fall 2021

This homework is due on Wed., Oct. 6. (Turn in your answers to questions 1–5.)
You may cite results from class, as appropriate.

0. *(This problem is NOT to be turned in.)*
   (a) Read Sections 4.3–4.6
   (b) Section 4.3 #4.31, 4.32
   (c) Section 4.4 #4.44
   (d) Section 4.5 #4.53
   (e) Section 4.6 #4.68

1. Use the triangle inequality to prove the following inequality for all real numbers x, y, z:

$$|x - z| \ \leq \ |x - y| + |y - z| \ .$$

2. Prove the following:
   For every real number x, if $|x| \geq 2$, then $x^2 \geq 4$.

3. Let $A = \{n \in \mathbb{Z} \mid n \equiv 1 (\text{mod } 2)\}$ and $B = \{n \in \mathbb{Z} \mid n \equiv 5 (\text{mod } 8)\}$.
   (a) Prove that $B \subseteq A$.
   (b) Is $B = A$? Explain your answer.

4. Suggest two problems for a future exam:
   - one from the Chapter 4 Supplementary Exercises (pg. 123), and
   - another one on any topic in Chapter 4 (please invent a problem, rather than taking one directly from the textbook).

5. (a) Section 4.3 #4.30
   (b) Section 4.4 #4.42
   (c) Section 4.5 #4.54
   (d) Section 4.6 #4.63 (and prove that your condition is indeed necessary and sufficient)