Homework 7

Math 300 (section 901), Fall 2021

This homework is due on Wed., Oct. 13. (Turn in your answers to questions 1–2.)
You may cite results from class, as appropriate.

0. *(This problem is NOT to be turned in.)*
   (a) Read Sections 5.1, 5.2, and 5.5
   (b) What is the difference between a proof by contradiction and a proof by contrapositive?
   (c) Is the product of two rational numbers, again a rational number?
   (d) Is the product of two irrational numbers, again an irrational number?
   (e) Section 5.1 #5.6
   (f) Section 5.2 #5.14, 5.15, 5.16
   (g) Section 5.5 #5.57

1. **Prove or disprove** the following:
   (a) The product of any irrational number and any nonzero rational number is irrational.
   (b) There is a smallest irrational number.
   (c) There is a largest rational number.
   (d) For an integer a, the following holds: $a^2|a$ if and only if $a = 0$ or $a = 1$ or $a = -1$.
   (e) For integers n, x, and y, if $n \nmid xy$, then $n \nmid x$ and $n \nmid y$.
   (f) For integers x and y, if $3|x$ and $5|y$, then $8|(x + y)$.
   (g) $\forall a \in \mathbb{Z},\ \forall b \in \mathbb{Z},\ 3|a \Rightarrow 9|(ab)$
   (h) $\exists a \in \mathbb{Z}\ , \exists b \in \mathbb{Z},\ a - b = 0.5$
   (i) For all sets A, B, and C, the following equality holds: $A \cap (B \cup C) = (A \cap B) \cup C$.
   (j) For integers a and b, if $7a + 3b$ is even, then a and b are of the same parity.
   (k) $\sqrt[3]{2}$ is irrational.

2. (a) Section 5.2 #5.20, 5.26
   (b) Section 5.5 #5.60