

# Homework 8

Math 300 (section 901), Fall 2021

This homework is due on Wed., October 20. (Turn in your answers to questions 1–5.) You may cite results from class, as appropriate.

0. (*This problem is NOT to be turned in.*)

- (a) Read Sections 5.3, 5.4, 6.1
- (b) Section 5.3 #5.36
- (c) Section 5.4 #5.45, 5.53

1. **Prove or disprove** the following:

- (a) There exist a rational number  $a$  and an irrational number  $b$  such that  $a^b$  is rational.
- (b) For every rational number  $x$ , there exists an integer  $n$  such that  $nx$  is an integer.
- (c) Every set can be written as the union of two sets.
- (d) There exists a real number  $x$  for which  $(x - 3)(x + 2) = 14$ .
- (e) Every even integer can be written as the sum of two multiples of 4.
- (f) Every even integers can be written as the sum of three even integers.
- (g) There exists an integer  $m$ , such that for all integers  $n \geq 2$ , the following congruence holds:  $m \equiv 1 \pmod{n}$ .
- (h) Let  $A$  denote the set of all multiples of 2, and let  $B$  denote the set of multiples of 2020. Then  $A = B$ .
- (i) The following containment holds:

$$\{a \in \mathbb{Z} \mid a \equiv 2 \pmod{4}\} \subseteq \{a \in \mathbb{Z} \mid a \equiv 2 \pmod{8}\} .$$

- (j) For every positive integer  $n$ , the following equality holds:

$$1 + 5 + 9 + \cdots + (4n - 3) = 2n^2 - n .$$

2. Section 5.4 #5.48(a)

3. Which part of Section 6.1 do you find to be the most interesting and/or surprising?

4. Is there something wrong with this supposed proof? If so, identify **all** the errors, and then either prove or disprove the claim. If not, explain why the proof is complete.

**Claim:** The average of three even numbers is an even number.

**Proof:** We proceed by contradiction: assume that the average of three even numbers is odd. However, the average of 2, 4, and 6, which is 4, is even. This is a contradiction.

5. Suggest two problems for a future exam:

- one from the Chapter 5 Supplementary Exercises, and
- another one on any topic in Chapter 5 (please invent a problem, rather than taking one directly from the textbook).

# Writing Assignment 4

Math 300, Fall 2021

This homework is due on Wednesday, October 27.

- Write a draft of three sections of your final paper:
  1. the introduction (what will your paper be about?),
  2. the mathematical background (define and/or explain all unfamiliar terms), and
  3. one section developing one of the main ideas from Writing Assignment 3.
- The expected length is at least two pages.
- *Please **print** 2 copies of your draft. As part of a future writing assignment, each student, plus the instructor or grader, will critique another student's draft.*
- If you do **not** turn in this draft, you will receive a 5% penalty on the final paper.