Homework 6

Math 669, Spring 2022

This homework is due on FRIDAY, March 11[1].

1. Consider the following chemical reaction network:

$$0 \leftrightarrows A \quad 2A \leftrightarrows 3A \ .$$

   (a) Write down the resulting mass-action ODE (ordinary differential equation).

   (b) What is the maximum number of positive steady states? What is the maximum number of *stable* positive steady states? Explain your answers.

   (c) Can this system exhibit *hysteresis*? If so, show this. If not, explain why not.

2. This problem concerns the article, *The core control system of intracellular iron homeostasis: A mathematical model*, by Chifman *et al.* (2012), available at `https://doi.org/10.1016/j.jtbi.2012.01.024`.

   (a) State one scientific question addressed in the article.

   (b) State one mathematical question addressed in the article.

   (c) Examine the 5 ODEs underlying their model (in Section 3.2). Is there a network for which the mass-action ODEs are those 5 ODEs? Explain.

   (d) What do the authors claim about the number of steady states and their stability? Do these depend on the parameters (α_i, γ_i, k_{ij}, etc.)?

   (e) Is your answer to (e) consistent with the biology (according to the article)?

3. Consider the following iron-trafficking model, which may be viewed as a much-simplified version of the Chifman *et al.* (2012) model:

$$\begin{aligned} \frac{dC}{dt} &= k_1 \left(\frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - k_2 C \left(1 - \frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - k_3 C - \alpha C \\ \frac{dF}{dt} &= k_2 C \left(1 - \frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - \alpha F \ , \end{aligned}$$

   where C represents iron in the cytosol and F represents ferritin, and n is a positive integer, $k_i > 0$ for all i, $\alpha > 0$, $T > 0$.

   (a) How does the number of steady states $(C, F) \in \mathbb{R}^2_{>0}$ depend on the values of n, k_i, T, α?

   (b) For each steady states, is it locally asymptotically stable? Does the stability depend on n, k_i, T, α?

   (c) *(Optional bonus problem)* Is this system globally asymptotically stable?

4. (Optional bonus problem) This problem concerns the "birthday chapter" introduction to chemical reaction networks (sent by email). List any typos you found, and provide any suggestions for improvement (e.g., which parts were confusing?).

[1] Turn in your homework to the instructor's office – Blocker 601-E – by 4:00 pm