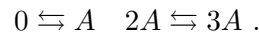


# Homework 6

Math 669, Spring 2022

This homework is due on FRIDAY, March 11<sup>1</sup>.

1. Consider the following chemical reaction network:



- (a) Write down the resulting mass-action ODE (ordinary differential equation).
  - (b) What is the maximum number of positive steady states? What is the maximum number of *stable* positive steady states? Explain your answers.
  - (c) Can this system exhibit *hysteresis*? If so, show this. If not, explain why not.
2. This problem concerns the article, *The core control system of intracellular iron homeostasis: A mathematical model*, by Chifman *et al.* (2012), available at <https://doi.org/10.1016/j.jtbi.2012.01.024>.
    - (a) State one scientific question addressed in the article.
    - (b) State one mathematical question addressed in the article.
    - (c) Examine the 5 ODEs underlying their model (in Section 3.2). Is there a network for which the mass-action ODEs are those 5 ODEs? Explain.
    - (d) What do the authors claim about the number of steady states and their stability? Do these depend on the parameters ( $\alpha_i$ ,  $\gamma_i$ ,  $k_{ij}$ , etc.)?
    - (e) Is your answer to (c) consistent with the biology (according to the article)?
  3. Consider the following iron-trafficking model, which may be viewed as a much-simplified version of the Chifman *et al.* (2012) model:

$$\begin{aligned} \frac{dC}{dt} &= k_1 \left( \frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - k_2 C \left( 1 - \frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - k_3 C - \alpha C \\ \frac{dF}{dt} &= k_2 C \left( 1 - \frac{1}{1 + \left(\frac{C}{T}\right)^n} \right) - \alpha F , \end{aligned}$$

where  $C$  represents iron in the cytosol and  $F$  represents ferritin, and  $n$  is a positive integer,  $k_i > 0$  for all  $i$ ,  $\alpha > 0$ ,  $T > 0$ .

- (a) How does the number of steady states  $(C, F) \in \mathbb{R}_{>0}^2$  depend on the values of  $n, k_i, T, \alpha$ ?
  - (b) For each steady states, is it locally asymptotically stable? Does the stability depend on  $n, k_i, T, \alpha$ ?
  - (c) (*Optional bonus problem*) Is this system globally asymptotically stable?
4. (*Optional bonus problem*) This problem concerns the “birthday chapter” introduction to chemical reaction networks (sent by email). List any typos you found, and provide any suggestions for improvement (e.g., which parts were confusing?).

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<sup>1</sup>Turn in your homework to the instructor’s office – Blocker 601-E – by 4:00 pm