Math 152 Week-in-Review

Exam 2 Review

1. Find $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$.

2. Evaluate $\displaystyle\int_0^{2/3} \frac{1}{(4+9x^2)^{5/2}}\,dx$

3. After choosing the appropriate trigonometric substitution for $\int (x-4)\sqrt{x^2-8x+7}\,dx$, write the resulting integral in terms of θ. Do NOT integrate.

4. Evaluate $\displaystyle\int \frac{x^4+2x^3-6x^2-2x+12}{x^3+4x^2+4x}\,dx$.

5. Evaluate $\displaystyle\int_0^2 \frac{x^2+3x}{(x+1)(x^2+4)}\,dx$.

6. Evaluate $\displaystyle\int_0^4 \frac{2}{(x-3)^2}\,dx$ or show it diverges.

7. Determine whether the improper integral converges or diverges using the comparison theorem.

   a.) $\displaystyle\int_1^\infty \frac{\sin(6x)+8}{\sqrt{x}+x^4}\,dx.$

b.) $\displaystyle\int_1^{\infty} \frac{e^{1/x}+2}{x}\,dx$

8. Find a general formula for the sequence $\left\{2, -\frac{7}{3}, \frac{12}{9}, -\frac{17}{27}, \frac{22}{81}, \cdots\right\}$. Assume the pattern continues, and the sequence begins with $n = 1$.

9. Consider the recursive sequence $a_1 = 4$ and $a_{n+1} = \dfrac{5}{6 - a_n}$.

(a) Find the first three terms of the sequence.

(b) Assuming the sequence is decreasing and bounded, determine if the sequence converges or diverges. If it converges, find the limit.

10. Determine whether the sequence converges or diverges. If it converges, what value does it converge to?

(a) $a_n = \sin n$

(b) $a_n = \dfrac{\sin n}{n}$

(c) $a_n = \cos\left(\dfrac{5}{n}\right)$

(d) $a_n = \dfrac{(-1)^n n}{2n^2+1}$

(e) $a_n = \dfrac{(-1)^{n-1}\, n}{2+9n}$

11. Determine if the following sequences are increasing, decreasing, or not monotonic. Also, determine if each sequence is bounded.

    (a) $a_n = 3 - e^{-2n}$

    (b) $a_n = (-1)^n n$

12. For the series $\displaystyle\sum_{n=1}^{\infty} a_n$, the n$th$ partial sum is given by $s_n = \dfrac{3-2n}{5n+1}$.

    (a) Find a_5

    (b) Find $\displaystyle\sum_{n=1}^{\infty} a_n$

    (c) What is $\displaystyle\lim_{n\to\infty} a_n$?

13. Determine if the following series converges or diverges. If the series converges, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty}\left[\frac{1}{2^n}-\frac{1}{2^{n+1}}\right]$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{6}{n(n+2)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{5 \cdot 3^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^3}\right)$

14. Determine whether the following series converge or diverge. Support your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{5}{n\sqrt{\ln n}}$

15. Using the remainder for the integral test, find an upper bound for the remainder if we use s_8 to approximate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

16. Using the remainder for the integral test, what is the smallest value of n that ensures s_n to approximates $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^4}$ with error less than $\dfrac{1}{100}$?

17. Determine if the following statements are true or false. If the statement is false, give a counter example

(a) If a sequence converges, then it is bounded.

(b) If a sequence is bounded, then it converges.

(c) If a sequence is increasing, then it converges.

(d) If $\lim_{n\to\infty} s_n = 4$, then $\sum_{n=1}^{\infty} a_n$ converges.

(e) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

(f) The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p \leq 1$.

(g) The geometric series $\sum_{n=2}^{\infty} ar^{n-1}$ converges if $|r| < 1$.