Math 152 Week-in-Review

Exam 3 Review

1. Determine if the series converges or diverges. FULLY explain your reasoning.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2+3\cos n}{n^3+4n^2}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{n+1}{5n^2-2}$

   (c) $\displaystyle\sum_{n=3}^{\infty} \frac{5+\sin n}{n-4\sqrt{n}}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$

2. Determine if the series converges absolutely, converges conditionally, or diverges. FULLY explain your reasoning.

a.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(3+\ln n)^3}$

b.) $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

c.) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n+1}$

d.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$

e.) $\sum_{n=1}^{\infty} \frac{(-10)^n n!}{(2n+3)!}$

3. Find the radius and interval of convergence for $\sum_{n=2}^{\infty} \frac{(x+3)^n}{5^n\sqrt{n-1}}$. FULLY explain your reasoning.

4. Find the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{(2x-3)^{n+1}n!}{100^n}$. FULLY explain your reasoning.

5. If $\sum_{n=0}^{\infty} c_n(x+2)^n$ converges at $x=5$, on what interval are we guaranteed convergence?

6. For the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+3)!}$, Use the Alternating Series Estimation Theorem to find an upper bound for the error if we used s_5 to estimate the sum.

7. Using The Alternating Series Estimation Theorem, what is the smallest value of n that guarantees s_n approximates $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+3}$ with error less than $\frac{1}{20}$?

8. Find a power series centered at 0 for the following functions:

   a.) $\dfrac{4}{6-x^2}$

   b.) $\dfrac{8x}{(6-x^2)^2}$, by using the result from above.

   c.) $\displaystyle\int x^4 \arctan(5x)\,dx$

9. Evaluate $\int_0^1 x^4 \ln(2 - x^3)\, dx$

10. Find $f^{(26)}(2)$ if $f(x) = \sum_{n=0}^{\infty} \frac{3^{n+1}(x-2)^n}{(n+8)!}$ is the Taylor Series for $f(x)$ centered at $a = 2$.

11. Find the Taylor Series centered at 4 for $f(x) = \frac{1}{(x+1)^2}$.

12. Find a Maclaurin series for e^{3x^2}.

13. Express $\int x^4 \cos(5x^3)\,dx$ as a power series about 0.

14. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{(2n+1)!}$

15. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-5)^n 2^{2n+1}}{n!}$

16. Find the third degree Taylor Polynomial for $f(x) = e^{-x}$ at $x = 2$.