Math 152 Week-in-Review

Final Exam Review Monday, May 1, 1-3pm HELD 100

1. Evaluate $\int_0^{(\pi^2/9)} \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$.

$u = \sqrt{x}$

$x = \frac{\pi^2}{9}, \; u = \frac{\pi}{3}$

$x = 0, \; u = 0$

$du = \frac{1}{2\sqrt{x}}$

$2\int_0^{\frac{\pi}{3}} \sin u \, du$

$-2\cos u \Big|_0^{\frac{\pi}{3}} = -2\left(\frac{1}{2} - 1\right)$

2. Find the area of the region bounded by $y = x^2 - 2x$ and $y = 2x - 3$.

$x^2 - 2x = 2x - 3$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

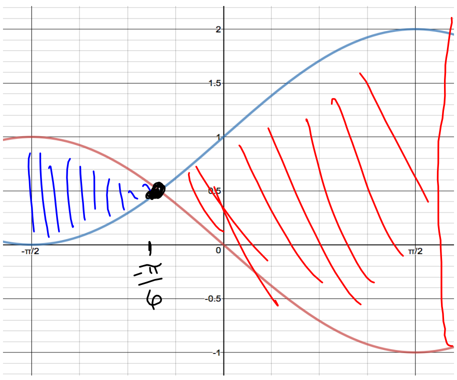
$$\boxed{\begin{array}{l} x = 3 \\ x = 1 \end{array}}$$

$x = 2$ $\begin{cases} y = x^2 - 2x \rightarrow y = 0 \leftarrow B \\ y = 2x - 3 \rightarrow y = 1 \leftarrow T \end{cases}$

$$A = \int_1^3 \left(2x - 3 - (x^2 - 2x)\right) dx$$

$= \underline{\qquad}$

3. Write an integral for the area of the region bounded by $y = -\sin x$ and $y = 1 + \sin x$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$. Do not evaluate.

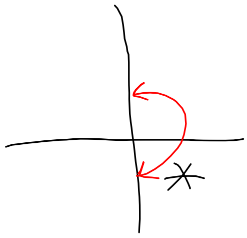


$$-\sin x = 1 + \sin x$$

$$-2\sin x = 1$$

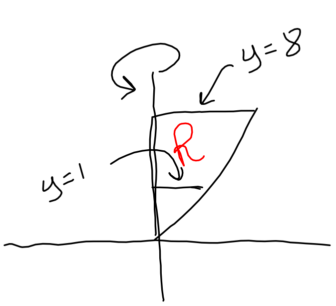
$$\sin x = -\frac{1}{2}$$

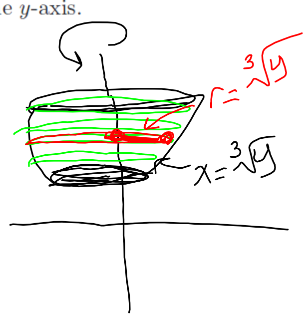
$$\boxed{x = -\frac{\pi}{6}}$$



$$A = \int_{-\frac{\pi}{2}}^{-\frac{\pi}{6}} \left(-\sin x - (1+\sin x)\right) dx + \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \left(1 + \sin x - (-\sin x)\right) dx$$

4. Consider the region bounded by $y = x^3$, $x = 0$, $y = 8$ and $y = 1$. Find the volume if this region is revolved around the y-axis.





disk → dy

$$V = \int_1^8 \pi r^2 \, dy$$

$$= \int_1^8 \pi \left(\sqrt[3]{y}\right)^2 dy$$

$$\pi \int_1^8 y^{\frac{2}{3}} \, dy$$

$$\pi \cdot \frac{3}{5} y^{\frac{5}{3}} \Big|_1^8$$

$$\frac{3\pi}{5}\left(\cancel{8^{\frac{5}{3}}} \, 32 - 1\right) = \boxed{\frac{3\pi}{5}(31)}$$

5. Consider the region bounded by $y = 4 - x^2$ and $y = -2x + 1$. Set up an integral, but do not evaluate, for the volume if the region is rotated around the following lines.

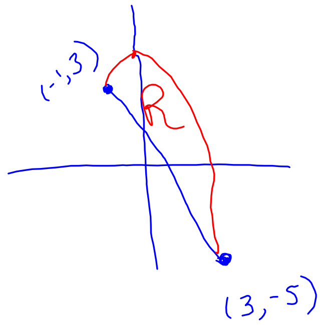
$4 - x^2 = -2x + 1$

$0 = x^2 - 2x - 3$

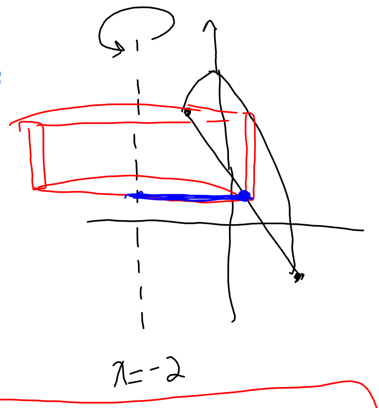
$0 = (x-3)(x+1)$

$x = 3 \longrightarrow y = -5$

$x = -1 \longrightarrow y = 3$



(a) $x = -2$



shell $\longrightarrow$ dx

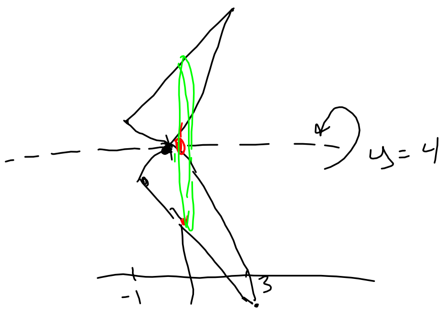
$V = \int 2\pi r h \, dx$

$h = T - B = 4 - x^2 - (-2x+1)$

$\boxed{h = 3 + 2x - x^2}$

$\boxed{r = x - (-2) = x + 2}$

$$\boxed{V = \int_{-1}^{3} 2\pi (x+2)(3+2x-x^2)}$$



(b) $y = 4$

washer $\longrightarrow$ dx

$$V = \int_{-1}^{3} \pi \left(R^2 - r^2\right) dx$$

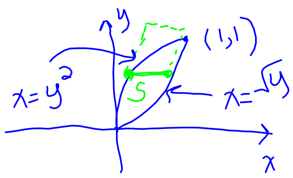
$R = 4 - (-2x+1) = 3 + 2x$

$r = 4 - (4 - x^2) = x^2$

$$V = \pi \int_{-1}^{3} \left((3+2x)^2 - x^4\right) dx$$

6. The solid S has a base bounded by the functions $y = x^2$ and $y = \sqrt{x}$. Cross-sections perpendicular to the $\boxed{y\text{-axis}}$ are $\boxed{\text{squares}}$. Set up an integral for the volume of S, but do not evaluate.

$y\text{-axis} \to dy$ squares $\to S^2$



$$V = \int \left(A_{\text{cross-section}}\right) dy$$

$$V = \int S^2\, dy$$

$$= \int_0^1 \left(\sqrt{y} - y^2\right)^2 dy$$

$y = x^2 \to x = \sqrt{y}$

$y = \sqrt{x} \to x = y^2$

$S = \sqrt{y} - y^2$

7. A spring has a natural length of 3 m. The force required to keep the spring stretched to a length of 8 m is 7 N. Find the work required to stretch the spring from a length of 4 m to a length of 6 m.

Hooke's Law $\to f(x) = kx$

$f(x)$ = force = 7 N

x = units beyond natural length

$7 = k(5) \to k = \frac{7}{5}$

$x = 8 - 3 = 5$

$f(x) = \frac{7}{5}x$

$$W = \int f(x)\,dx = \int_{4-3}^{6-3} \frac{7}{5}x\,dx = \int_1^3 \frac{7}{5}x\,dx$$

8. A rope that weighs $\frac{1}{6}$ pounds per foot is used to pull a bucket full of water to the top of a 60 ft well. The bucket weighs 15 pounds when full of water. How much work is done?

$$W = \int_0^{\text{How much of rope pulling}} \left(\cancelto{25}{\text{Total weight}} - \left(\cancelto{\frac{1}{6}}{\text{weight of rope per unit}}\right) y\right) dy$$

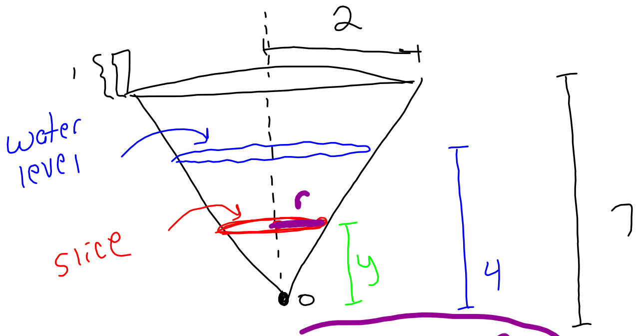
$$\text{Total weight} = \left(\frac{1}{6}\,\frac{\text{lbs}}{\text{foot}}\right)(60\text{ ft}) + 15\text{ lbs}$$

$$= 25\text{ lbs}$$

$$\text{weight of rope per unit} = \frac{1}{6}\,\frac{\text{lbs}}{\text{ft}}$$

$$W = \int_0^{60} \left(25 - \frac{1}{6}y\right) dy$$

9. A tank has the shape of a cone, where the radius at the top is 2 m, and the cone is 7 m high. Suppose the water in the tank is 4 m deep, and all the water is pumped out of a spout 1 m above the top of the cone. Write an integral for how much work is done, but do not evaluate. Use ρg for the weight density of water.



$$W = \int_0^{\text{water level}} (\rho g)(V_{\text{slice}})\, d$$

$$d = \text{height of tank} + \text{spout} - y$$

$$\boxed{d = 7 + 1 - y = 8 - y}$$

$$V = \pi r^2 dy \longrightarrow v = \pi\left(\frac{4}{49} y^2\right) dy$$

$$\frac{r}{y} = \frac{2}{7} \longrightarrow r = \frac{2}{7} y$$

$$W = \int_0^4 \rho g\left(\pi \frac{4}{49} y^2\right)(8 - y)\, dy$$

10. Integrate the following

(a) $\displaystyle\int_0^5 \frac{x}{\sqrt{9-x}}\,dx$

$u = 9 - x$; $x = 5,\ u = 4$; $x = 0,\ u = 9$

$du = -dx$

$x = 9 - u$

$$-\int_9^4 \frac{9-u}{\sqrt{u}}\,du = -\int_9^4 \left(9u^{-\frac{1}{2}} - u^{\frac{1}{2}}\right) du$$

$$= \int_4^9 \left(9u^{-\frac{1}{2}} - u^{\frac{1}{2}}\right) du$$

(b) $\displaystyle\int \sqrt{x}\ln(x)\,dx$

Parts $\quad \int u\,dv = uv - \int v\,du$

$u =$

L $\ln x$

I inverse trig

P polynomial

E e^x

T trig

$u = \ln x \qquad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{2}{3}x^{\frac{3}{2}}$

$$\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{\frac{3}{2}}\ln x - \int \frac{2}{3}x^{\frac{3}{2}}\frac{1}{x}\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,dx$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{\frac{3}{2}} + C$$

(c) $\int_0^{\pi/6} \sin^2(2x)\,dx.$ $= \int_0^{\frac{\pi}{6}} \frac{1}{2}\left(1-\cos(4x)\right)dx$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\frac{1}{2}\left(x - \frac{1}{4}\sin(4x)\right)\Big|_0^{\frac{\pi}{6}}$

$\frac{1}{2}\left(\frac{\pi}{6} - \frac{1}{4}\underbrace{\sin\left(\frac{2\pi}{3}\right)}_{\frac{\sqrt{3}}{2}} - 0\right)$

(d) $\int \sin^3(x)\cos^2(x)\,dx$ $= \int \underbrace{\sin^2 x}_{1-\cos^2 x = 1-u^2}\ \underbrace{\cos^2 x}_{u^2}\ \underbrace{\sin x\,dx}_{-du}$

$u = \cos x$

$du = -\sin x\,dx$

$= -\int(1-u^2)u^2\,du$

$\vdots$

(e) $\int \tan^3(x)\sec^5(x)\,dx = \int \underbrace{\tan^2 x}_{\sec^2 x - 1 \to u^2-1} \cdot \underbrace{\sec^4 x}_{u^4} \underbrace{\sec x \tan x\, dx}_{du}$

$u = \sec x$

$du = \sec x \tan x\, dx$

$$\int (u^2-1)u^4\,du$$

(f) $\int_0^1 \frac{x^2}{e^x}\,dx = \int_0^1 x^2 e^{-x}\,dx$

Tabular!!

u	dv
x^2 (+)	e^{-x}
$2x$ (−)	$-e^{-x}$
2 (+)	e^{-x}
0	$-e^{-x}$

$$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\Big|_0^1$$

$$= -e^{-1} - 2e^{-1} - 2e^{-1} - (-2e^0)$$

$$= -5e^{-1} + 2$$

(g) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-16}}$

$x^2 - a^2 \rightarrow x = a\sec\theta$

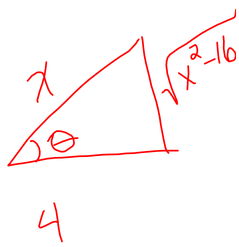
$x = 4\sec\theta$

$dx = 4\sec\theta\tan\theta\, d\theta$

$$\int \frac{4\sec\theta\tan\theta\, d\theta}{16\sec^2\theta\sqrt{\underbrace{16\sec^2\theta - 16}_{16(\sec^2\theta-1)=16\tan^2\theta}}}$$

$$\int \frac{\cancel{4}\cancel{\sec\theta}\cancel{\tan\theta}\, d\theta}{16\sec^{\cancel{2}}\theta\cdot\cancel{4}\cancel{\tan\theta}} = \frac{1}{16}\int\frac{d\theta}{\sec\theta}$$

$\frac{H}{A}\ \frac{x}{4} = \sec\theta$



$$= \frac{1}{16}\int \cos\theta\, d\theta$$

$$= \frac{1}{16}\sin\theta + C$$

$$= \boxed{\frac{1}{16}\,\frac{\sqrt{x^2-16}}{x} + C}$$

(h) $\int_0^{1/3} \sqrt{4-9x^2}\, dx$

(i) $\int \frac{x^3 - 6x^2 + 7x - 9}{x^2 - 3x}\, dx$

D Q

$$\begin{array}{r} x - 3 \\ x^2 - 3x \,\overline{)\, x^3 - 6x^2 + 7x - 9} \\ -\;x^3 + 3x^2 \\ \hline -3x^2 + 7x - 9 \\ +3x^2 - 9x \\ \hline -2x - 9 \end{array}$$

R

PFD

$$\int \left(x - 3 + \frac{-2x - 9}{x^2 - 3x} \right) dx$$

$$\frac{-2x-9}{x(x-3)} = \frac{\cancel{A}^{\,3}}{x} + \frac{\cancel{B}^{\,-5}}{x-3}$$

$$-2x - 9 = A(x-3) + Bx$$

$x = 0: \quad -9 = A(-3)$

$\boxed{A = 3}$

$x = 3: \quad -15 = B(3)$

$\boxed{B = -5}$

$$\int \left(x - 3 + \frac{3}{x} - \frac{5}{x-3} \right) dx$$

$$\boxed{\frac{x^2}{2} - 3x + 3 \ln|x| - 5 \ln|x-3| + C}$$

(j) $\int \frac{8x + 12}{x^4 + 4x^2}\, dx$

PFD: $\frac{8x+12}{x^2(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$

(k) $\int_1^\infty 5x^2 e^{-x^3}\,dx. \quad = \frac{-5}{3}\int e^{u}\,du$

$u = -x^3$
$du = -3x^2\,dx$

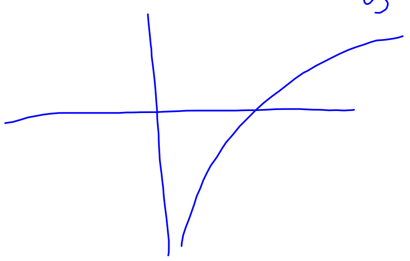
$= -\frac{5}{3} e^{-x^3}\Big|_1^\infty$

$= -\frac{5}{3}\left[\cancel{e^{-\infty}}^{\to 0} - e^{-1} \right]$

$= \frac{-5}{3}\left(-e^{-1}\right)$ $\boxed{\text{converges to } \frac{5}{3e}}$

(1) $\int_0^5 \frac{1}{2x-5}\,dx = \int_0^{\frac{5}{2}} \frac{1}{2x-5}\,dx + \int_{\frac{5}{2}}^5 \frac{1}{2x-5}\,dx$

$\frac{1}{2}\ln|2x-5| \Big|_0^{\frac{5}{2}}$



$\frac{1}{2}\left[\overset{-\infty}{\ln(0)} - \ln 5\right] =$

$\boxed{\text{original diverges}}$

11. Determine if the following integral converges or diverges: $\displaystyle\int_3^\infty \frac{5-\cos(x)}{\sqrt{x-2}}\,dx$

12. Sketch the parametric curves described below.

a.) $x = t^2$, $y = t + 5$

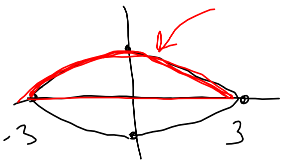
b.) $x = 3\cos\theta$, $y = 2\sin\theta$, $\boxed{0 \le \theta \le \pi}$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{x}{3} = \cos\theta$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{y}{2} = \sin\theta$



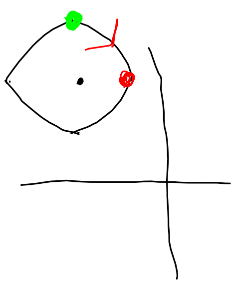
13. What is the shape of the curve $x = -2 + \sin t$, $y = 5 + \cos t$? In what direction is the curve traced as t increases?

$\sin t = x + 2$

$\sin^2 t + \cos^2 t = 1$

$\cos t = y - 5$

$(x+2)^2 + (y-5)^2 = 1$



$\boxed{\text{circle}}$ $\boxed{\text{clockwise}}$

$t = 0$: $x = -2$, $y = 6$

$t = \frac{\pi}{2}$: $x = -1$, $y = 5$

14. Find the length of the curve $x = 2t^2 + \frac{1}{t}$, $y = 8\sqrt{t}$, $1 \le t \le 3$.

$$L = \int_1^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

15. Setup an integral, but do not evaluate, for the the surface area obtained by rotating the curve $x = 3t\cos\left(\frac{t}{2}\right)$ and $y = 2e^{t/3}$, $0 \le t \le \frac{\pi}{2}$, about the x-axis, then about the y-axis.

$\hookrightarrow r = x = 3t\cos\left(\frac{t}{3}\right)$

$\hookrightarrow r = y = 2e^{\frac{t}{3}}$

$$SA = \int_0^{\frac{\pi}{2}} 2\pi r \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

16. Write two polar representations, one with $r > 0$ and one with $r < 0$ for the point $(-5\sqrt{3}, 5)$.

θ II

Polar point: (r, θ)

$x = -5\sqrt{3}$

$y = 5$

$x^2 + y^2 = r^2$

$25(3) + 25 = r^2$

$r = \pm 10$

$r > 0 \rightarrow r = 10, \ \theta = \frac{5\pi}{6}$ $\boxed{\left(10, \frac{5\pi}{6}\right)}$

$\tan\theta = \frac{y}{x} = \frac{5}{-5\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$\theta = \frac{5\pi}{6}$ $\boxed{\left(-10, \frac{11\pi}{6}\right)}$

$r < 0 \rightarrow r = -10$

$\theta = \frac{11\pi}{6}$ or $\frac{-\pi}{6}$

17. Write a polar equation for the cartesian equation $x^2 + y^2 = 5x$. What is the shape of the graph?

$x^2 + y^2 = r^2$

$x = r\cos\theta$

$r = r\sin\theta$

$r^2 = 5r\cos\theta$

$r = 5\cos\theta$

circle

$$\boxed{\begin{array}{ll} r = a\cos\theta & \text{both} \\ r = a\sin\theta & \text{circles} \end{array}}$$

18. Find a cartesian equation for $r = 2\csc\theta$. What is the shape of the graph?

$r = \frac{2}{\sin\theta}$

$r\sin\theta = 2$

$y = 2$ Line!

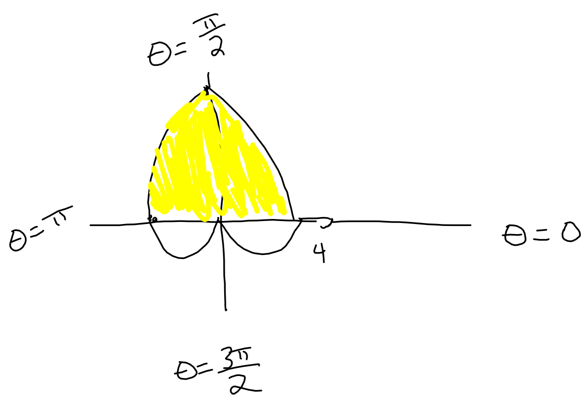
19. Find the area inside the cardioid $r = 4 + 4\sin\theta$ that lies above the x-axis.

Area of polar curve $r = f(\theta)$, $\alpha \le \theta \le \beta$

$$A = \int_\alpha^\beta \frac{1}{2} r^2 \, d\theta$$

generate the whole cardioid

$$V = \int_0^{2\pi} \frac{1}{2}(4 + 4\sin\theta)^2 \, d\theta$$



$r = 4 + 4\sin\theta$

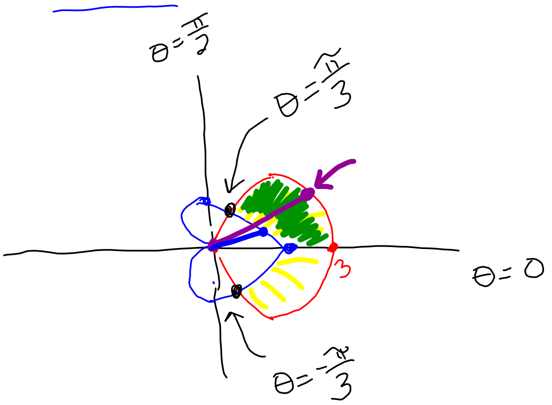
$$\boxed{\begin{array}{l} \theta = 0, \quad r = 4 \\ \theta = \frac{\pi}{2}, \quad r = 8 \\ \theta = \pi, \quad r = 4 \end{array}}$$

$\theta = \frac{3\pi}{2}, \quad r = 0$

$$A = \int_0^{\pi} \frac{1}{2}(4 + 4\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \left(16 + 8\sin\theta + 16\underbrace{\sin^2\theta}_{\frac{1}{2}(1-\cos 2\theta)}\right) d\theta$$

20. Find the area of the region inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$.



$3\cos\theta = 1 + \cos\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

symmetry: $A = 2\int_0^{\frac{\pi}{3}} \frac{1}{2}\left[(3\cos\theta)^2 - (1+\cos\theta)^2\right] d\theta$

21. Find the area of one petal of the rose $r = 2\cos(3\theta)$.

$\cos(k\theta)$ or $\sin(k\theta)$

k = odd, k petals $\longrightarrow$ whole graph generated for $0 \le \theta \le \pi$

k = even, $2k$ petals $\longrightarrow$ whole graph generated for $0 \le \theta \le 2\pi$

$k = 3 =$ odd, 3 petals

Area of one petal $= \dfrac{\text{Total enclosed area}}{3}$

$$= \int_0^{\pi} \frac{\frac{1}{2}(2\cos 3\theta)^2}{3} \, d\theta$$

$$= \frac{1}{3} \int_0^{\pi} \frac{1}{2}\left(4\cos^2 3\theta\right) d\theta$$

22. Determine if the sequence $a_n = \ln(2n + 3e^{-n}) - \ln(5n + 2)$ converges or diverges. If it converges, find its limit.

23. Determine if the sequence $a_n = \dfrac{(-1)^n(5n - n^3)}{2n^3 + 3n^2}$ converges or diverges. If it converges, find its limit.

24. Suppose that $s_n = \dfrac{2n-3}{n+4}$ is the sequence of partial sums for the series $\sum_{n=1}^{\infty} a_n$.
Find a_4 and $\sum_{n=1}^{\infty} a_n$.

25. Given the recursive sequence $a_1 = 4$ and $a_{n+1} = 10 - \dfrac{21}{a_n}$ is bounded and increasing, find the limit.

26. Find a general formula for the sequence $\left\{ -\dfrac{7}{4}, \dfrac{12}{9}, -\dfrac{17}{16}, \dfrac{22}{25}, -\dfrac{27}{36}, \dots \right\}$. Assume the pattern continues and starts with $n = 1$.

27. Determine if the series $\sum_{n=1}^{\infty}\left[e^{1/n}-e^{1/(n+2)}\right]$ converges or diverges. If it converges, find its sum.

$\underline{n} \quad n+1 \quad \underline{n+2}$

$S_n = a_1 + a_2 + a_3 + \cdots a_{n-1} + a_n$

$S_n = e - \cancel{e^{\frac{1}{3}}} + e^{\frac{1}{2}} - \cancel{e^{\frac{1}{4}}} + \cancel{e^{\frac{1}{3}}} - \cancel{e^{\frac{1}{5}}} + \cancel{e^{\frac{1}{4}}} - \cancel{e^{\frac{1}{6}}} + \cdots + \cancel{e^{\frac{1}{n-1}}} - e^{\frac{1}{n+1}} + \cancel{e^{\frac{1}{n}}} - e^{\frac{1}{n+2}}$

$S_n = e + e^{\frac{1}{2}} - e^{\frac{1}{n+1}} - e^{\frac{1}{n+2}}$

$\lim_{n\to\infty} S_n = \boxed{e + e^{\frac{1}{2}} - 2}$

28. Determine if the series converges absolutely, converges but not absolutely, or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{\sqrt{n}}$$

29. Determine if the series converges or diverges and justify your answer.

$$\sum_{n=3}^{\infty} \frac{\cos^2(n)+4}{n-2}$$

30. Determine if the series converges or diverges. If it converges, find its sum

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4^n}{7^{n-1}}$$

31. Determine if the series converges absolutely, converges but not absolutely, or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n}}{n^3+2n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\left(\sqrt{n}-3\right)}{n+7}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+2n)}{4n^2-3n}$

32. Determine if the series $\sum_{n=1}^{\infty} \frac{e^{1/n^2}}{n^3}$ converges or diverges.

33. Determine whether $\sum_{n=1}^{\infty} \frac{n^3 3^{2n}}{(-5)^{n+1}}$ converges or diverges.

34. Determine if the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\arctan n}{3 + 2e^{-n}}$$

35. Find the radius and interval of convergence for the power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1}(2x-5)^n}{3^n \ln n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 7^{n-1}(x+3)^n}{n^4(n+2)!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(4x-7)^n}{e^n\sqrt{n}}$

36. Suppose that the series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges when $x=-4$ and diverges when $x=6$.
What can be said about the convergence of the following series?
(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n c_n 3^n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n 4^n$ (c) $\displaystyle\sum_{n=0}^{\infty} c_n 5^n$ (d) $\displaystyle\sum_{n=0}^{\infty} c_n(-6)^n$ (e) $\displaystyle\sum_{n=0}^{\infty} c_n 9^n$

37. Write the Maclaurin series for the function $\dfrac{5x^4}{5+2x^2}$.

38. Write the Maclaurin series for the function $\ln(6-2x^3)$.

39. Write the Maclaurin series for the function $\dfrac{x^2}{(7+x)^2}$.

40. Write the Maclaurin series for $3x^2 \cos\left(\frac{x^3}{3}\right)$.

41. Write the Maclaurin series for $x^3 e^{-2x^2}$.

42. Calculate $\int \sin\left(3x^4\right)\, dx$ as a Maclaurin series.

43. Find the sum of the following series: $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n} \pi^{2n}}{6^{2n}(2n)!}$

44. Find the Taylor series for the function $f(x) = \frac{2}{x^2}$ centered at $a = -5$.

45. Find the second degree Taylor polynomial, $T_2(x)$, for $f(x) = \cos(2x)$ centered at $a = \frac{\pi}{6}$.