Fall 2019 Math 152

Week in Review I

courtesy: Amy Austin

(covering sections 5.5-6.1)

Section 5.5

1. $\int \frac{1+x^2-x}{\sqrt{x}}\,dx =$

$$\int (1+x^2-x)\,x^{-1/2}\;dx$$

$$\int \frac{1}{\sqrt{x}} + \frac{x^2}{\sqrt{x}} - \frac{x}{\sqrt{x}}\;dx$$

$$\int x^{-1/2} + x^{3/2} - x^{1/2}\;dx$$

$$\boxed{2x^{1/2} + \frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C}$$

2. $\int_0^1 (x^3 - 2)^2\, dx =$

$$\int_0^1 (x^3-2)(x^3-2)\, dx$$

$$\int_0^1 x^6 - 4x^3 + 4 \quad dx$$

$$\frac{1}{7}x^7 - \frac{4}{4}x^4 + 4x \Big|_0^1$$

$$\left[\frac{1}{7}(1)^7 - (1)^4 + 4(1)\right]$$

$$-\cancel{\left[\frac{1}{7}(0)^7 - (0)^4 + 4(0)\right]} \quad 0$$

$$= \frac{1}{7} - 1 + 4 = 3 + \frac{1}{7} = \boxed{\frac{22}{7}}$$

3. $\int 5x^2(3x^3-1)^8\,dx =$

$u = 3x^3 - 1$

$du = 9x^2\,dx$

Alternative method : $dx = \dfrac{du}{9x^2}$

$$\frac{5}{9}\int 9x^2\,(3x^3-1)^8\,dx$$

$$\frac{5}{9}\int u^8\,du$$

$$\frac{5}{9}\cdot\frac{1}{9}u^9 + C = \boxed{\frac{5}{81}(3x^3-1)^9 + C}$$

$$4.\ \frac{1}{6}\int_{x=0}^{x=1} 6x^2 e^{2x^3-5}\,dx =$$

$u = 2x^3 - 5$

$du = 6x^2\,dx$

$u(1) = 2(1)^3 - 5 = -3$

$u(0) = 2(0)^3 - 5 = -5$

$$\frac{1}{6}\int_{-5}^{-3} e^{u}\,du$$

$$\frac{1}{6} e^{u} \Big|_{-5}^{-3} = \boxed{\left[\frac{1}{6} e^{-3}\right] - \left[\frac{1}{6} e^{-5}\right]}$$

$$5.\ \frac{1}{-2}\int_{-4}^{0} \frac{1}{\sqrt{1-2x}} \cdot (-2)\, dx =$$

$u = 1-2x$

$du = -2\ dx$

Bounds:

$u(0) = 1-2(0) = 1$

$u(-4) = 1-2(-4) = 9$

$$\frac{-1}{2}\int_{9}^{1} \frac{1}{\sqrt{u}}\, du = \frac{1}{2}\int_{1}^{9} u^{-1/2}\, du$$

$$\frac{1}{2}\cdot 2u^{1/2}\Big|_{1}^{9} = \sqrt{9}-\sqrt{1} = 3-1 = \boxed{2}$$

$$\frac{-1}{2}\int \frac{1}{\sqrt{u}}\, du = \frac{-1}{2}\int u^{-1/2}\, du = \frac{-1}{2}\cdot 2u^{1/2} = -(1-2x)\Big|_{-4}^{0}$$

$$6.\ \frac{1}{\pi}\int_1^{1/2} \pi \cdot \cos(\pi x)\,dx =$$

$u = \pi x$

$du = \pi\ dx$

Bounds

$u\left(\frac{1}{2}\right) = \pi\ \frac{1}{2} = \frac{\pi}{2}$

$u(1) = \pi(1) = \pi$

$$\frac{1}{\pi}\int_{\pi}^{\pi/2} \cos(u)\ du = \frac{1}{\pi}\ \sin(u)\Big|_{\pi}^{\pi/2}$$

$$\left[\frac{1}{\pi}\sin\left(\frac{\pi}{2}\right)\right] - \left[\frac{1}{\pi}\ \sin(\pi)\right]$$

$$\frac{1}{\pi}\cdot 1 - \frac{1}{\pi}\cdot 0 = \boxed{\frac{1}{\pi}}$$

7. $\displaystyle\int_0^{\pi/12} \tan(3x)\,dx =$

$$\frac{-1}{3}\int_0^{\pi/12} \frac{-3\sin(3x)}{\cos(3x)}\,dx$$

$u = \cos(3x)$

$du = -3\sin(3x)\,dx$

Bounds:

$u\left(\frac{\pi}{12}\right) = \cos\left(\frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$u(0) = \cos(0) = 1$

$$\frac{-1}{3}\int_1^{\frac{\sqrt{2}}{2}} \frac{1}{u}\,du$$

$$\frac{1}{3}\int_{\frac{\sqrt{2}}{2}}^{1} \frac{1}{u}\,du = \frac{1}{3}\ln|u|\Big|_{\frac{\sqrt{2}}{2}}^{1}$$

$$\underbrace{\frac{1}{3}\ln(1)}_{=0} - \frac{1}{3}\ln\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{-1}{3}\ln\left(\frac{\sqrt{2}}{2}\right)}$$

$$8. \int \left(\frac{1}{\sqrt{1-x^2}} - 4x^{-1} + e^x + \frac{2}{x^2+1} - \frac{x+1}{x^2+4} \right) dx =$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx \qquad \int -4 \cdot \frac{1}{x}\,dx \qquad \int e^x\,dx \qquad 2\int \frac{1}{x^2+1}\,dx \qquad -\frac{1}{2}\int \frac{2x}{x^2+4}\,dx \qquad -\int \frac{1}{x^2+4}\,dx$$

$$\boxed{\arcsin(x) - 4\ln|x| + e^x + 2\arctan(x) - \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$

$u = x^2+4$

$du = 2x\,dx$

$$-\frac{1}{2}\int \frac{1}{u}\,du$$

$$-\frac{1}{2}\ln|u|$$

$$-\frac{1}{2}\ln|x^2+4|$$

$$\frac{1}{x^2+4} = \frac{1}{4\left(\frac{x^2}{2^2}+1\right)}$$

$$\frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}$$

9. $\int \frac{e^x}{1+e^x}\,dx =$

$u = 1+e^x$

$du = e^x\,dx$

$$\int \frac{1}{u}\,du = \ln|u| = \boxed{\ln|1+e^x| + C}$$

$$\boxed{\ln(1+e^x) + C}$$

$\cancel{\ln(1) \quad \ln(1)}$

$\cancel{\ln(1+1) = \ln(2)}$

$$10.\ \frac{5}{2}\int_1^2 \frac{2}{2x+1}\,dx =$$

$u = 2x+1$

$du = 2\,dx$

Bounds:

$u(2) = 2(2)+1 = 5$

$u(1) = 2(1)+1 = 3$

$$\frac{5}{2}\int_3^5 \frac{1}{u}\,du = \frac{5}{2}\ln|u|\Big|_3^5$$

$$\frac{5}{2}\left[\ln(5) - \ln(3)\right]$$

$$\boxed{\frac{5}{2}\ln\left(\frac{5}{3}\right)}$$

11. $-\int \frac{-\sin t}{\cos^5 t}\,dt =$

$\cos^5(t) = [\cos t]^5$

$u = \cos(t)$

$du = -\sin(t)\;dt$

$$-\int \frac{1}{u^5}\,du = -\int u^{-5}\,du = -\cdot\frac{-1}{4}u^{-4} + C = \frac{1}{4}\cdot\frac{1}{u^4} + C$$

$$\boxed{\frac{1}{4\cos^4(t)} + C}$$

12. $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx =$

$u = x+1 \Rightarrow x = u-1$

$du = dx$

$$\int \frac{u-1}{\sqrt{u}}\,du = \int u^{1/2} - u^{-1/2}\,du$$

$$\frac{2}{3}u^{3/2} - 2u^{1/2} + C = \boxed{\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C}$$

$$13.\ \int \frac{2x^3}{x^2-1}\,dx =$$

$u = x^2-1 \Rightarrow u+1 = x^2$

$du = 2x\ dx$

$$\frac{2}{2}\int \frac{x^2 \cdot 2x}{x^2-1}\ dx$$

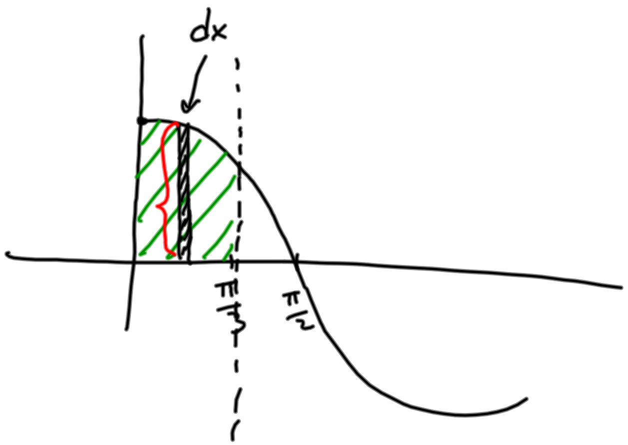
$$\int \frac{u+1}{u}\,du = \int 1 + \frac{1}{u}\ du = u + \ln|u| + C$$

$$\boxed{x^2-1 + \ln|x^2-1| + C}$$

Section 6.1

14. Find the area bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \frac{\pi}{3}$.

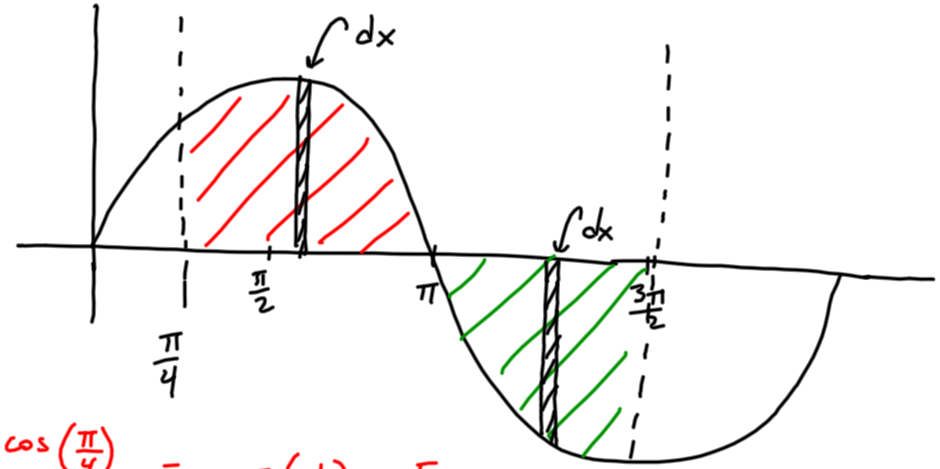
$$\int_0^{\pi/3} [\cos(x) - 0]\, dx$$



$$\sin(x)\Big|_0^{\pi/3} = \sin\left(\frac{\pi}{3}\right) - \cancel{\sin(0)}$$

$$\boxed{\frac{\sqrt{3}}{2}}$$

15. Find the area bounded by $y = \sin x$, $y = 0$, $x = \dfrac{\pi}{4}$, $x = \dfrac{3\pi}{2}$.



$$\int_{\frac{\pi}{4}}^{\pi} [\sin(x) - 0]\, dx = -\cos(x)\Big|_{\frac{\pi}{4}}^{\pi}$$

$$-\cos(\pi) + \cos\left(\frac{\pi}{4}\right) = -(-1) + \frac{\sqrt{2}}{2} = 1 + \frac{\sqrt{2}}{2}$$

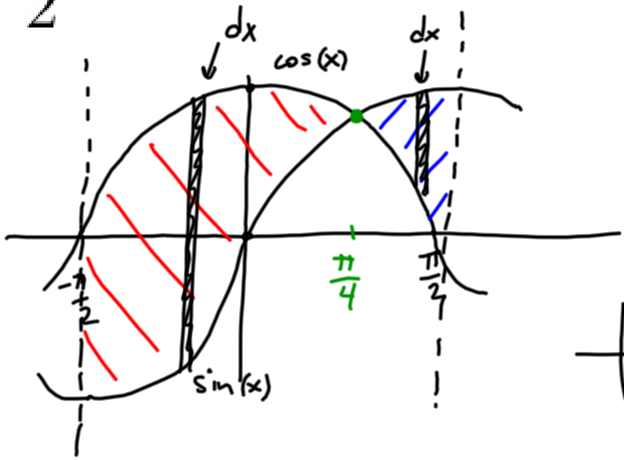
$$+ \int_{\pi}^{\frac{3\pi}{2}} [0 - \sin x]\, dx = \cos(x)\Big|_{\pi}^{\frac{3\pi}{2}} = \cos\left(\frac{3\pi}{2}\right) - \cos(\pi) = 0 - (-1) = 1$$

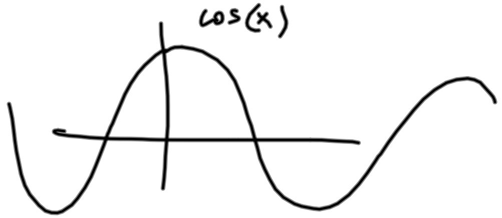
$$\text{Total Area} = 1 + \frac{\sqrt{2}}{2} + 1 = \boxed{2 + \frac{\sqrt{2}}{2}}$$

16. Find the area bounded by $y = \sin x$, $y = \cos x$, $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} [\cos(x) - \sin(x)]\, dx$$

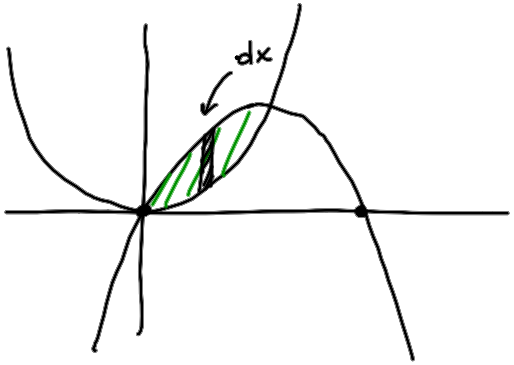
$$+ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [\sin(x) - \cos(x)]\, dx$$





$\cos(x) = \sin(x)$

17. Find the area bounded by $y = x^2$ and $y = 2x - x^2$.



$$0 = 2x - x^2$$

$$0 = x(2-x)$$

$$x=0 \qquad 2-x=0$$

$$x = 2$$

$$x^2 = 2x - x^2$$

$$2x^2 = 2x$$

$$2x^2 - 2x = 0$$

$$2x(x-1) = 0$$

$$x=0 \qquad x=1$$

$$\int_0^1 (2x-x^2) - (x^2)\, dx$$

$$\int_0^1 2x - 2x^2 \, dx$$

$$x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$\left[1 - \frac{2}{3}\right] - [0] = \boxed{\frac{1}{3}}$$

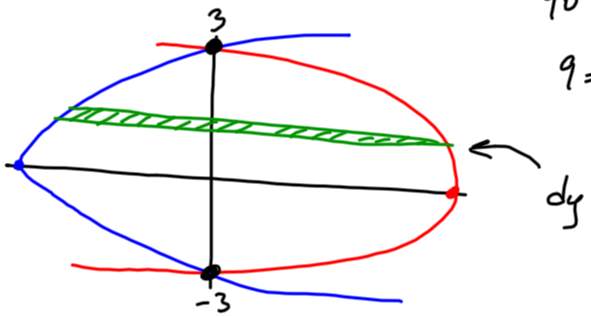
18. Find the area bounded by $x = 45 - 5y^2$ and $x = 5y^2 - 45$.

$45 - 5y^2 = 5y^2 - 45$

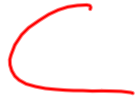
$90 = 10y^2$

$9 = y^2$

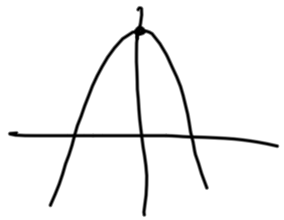
$y = \pm 3$



$x = y^2$



$y = 45 - 5x^2$



$$\boxed{\int_{-3}^{3} \left[(45 - 5y^2) - (5y^2 - 45) \right] dy}$$

or $\quad 2 \displaystyle\int_0^3 (45 - 5y^2) - (5y^2 - 45)\, dy$

or $\quad 4 \displaystyle\int_0^3 (45 - 5y^2)\, dy$

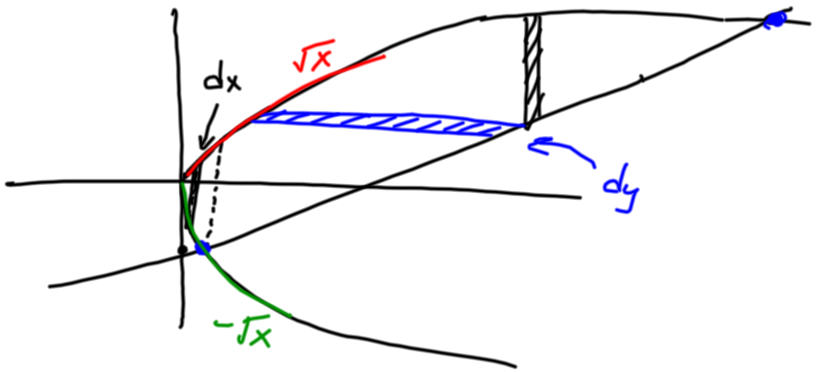
19. Sketch the region R bounded by $x = y^2$ and $x = 5y + 6$. Set up but do not evaluate an integral in terms of y and then an integral in terms of x that gives the area of this region.

$x - 6 = 5y$

$y = \frac{x}{5} - \frac{6}{5}$

$x = y^2$

$y = \pm\sqrt{x}$



$$\boxed{\int_{-1}^{6} (5y+6) - y^2 \, dy}$$

$y^2 = 5y + 6$

$y^2 - 5y - 6 = 0$

$(y-6)(y+1) = 0$

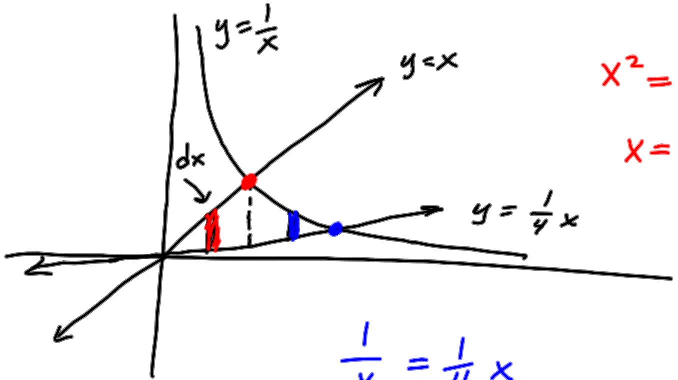
$y = 6$

$y = -1$

$$\boxed{\int_0^1 \sqrt{x} - (-\sqrt{x}) \, dx + \int_1^{36} \sqrt{x} - \left(\frac{x}{5} - \frac{6}{5}\right) dx}$$

20. Sketch the region R bounded by $y = \frac{1}{x}$, $y = x$, $y = \frac{1}{4}x$, $x \geq 0$. Set up but do not evaluate an integral that gives the area of R.

$$\boxed{\int_0^1 \left(x - \frac{1}{4}x\right)\,dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right)\,dx}$$



$x = \frac{1}{x}$

$x^2 = 1$

$x = \pm 1$

$\frac{1}{x} = \frac{1}{4}x$

$4 = x^2$

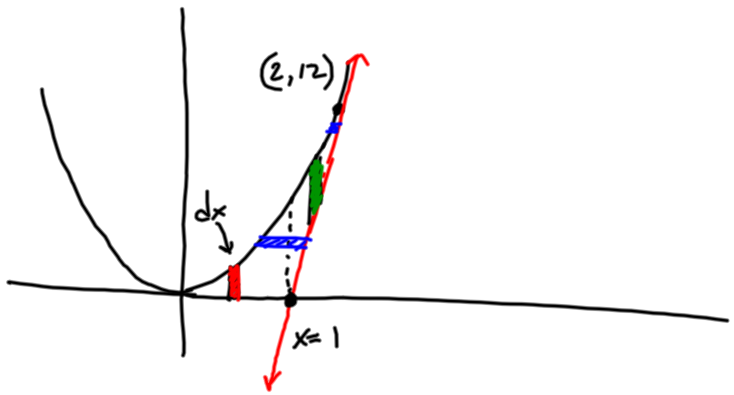
$x = \pm 2$

21. Find the area of the region bounded by the parabola $y = 3x^2$, the tangent line to this parabola at $(2, 12)$ and the x-axis.

$\frac{dy}{dx} = 6x$

@ $x = 2$

$m = 12$



Line: $y - 12 = 12(x-2)$

$y - 12 = 12x - 24$

$y = 12x - 12$

$$\boxed{\int_0^1 3x^2 - 0 \, dx + \int_1^2 3x^2 - (12x - 12) \, dx}$$

$12x - 12 = 0 \qquad 12x = 12 \qquad x = 1$