MATH 251 Fall 2018
EXAM III - VERSION A

LAST NAME: Key FIRST NAME: ____________

SECTION NUMBER: ____________

UIN: ____________

DIRECTIONS:

1. This is a non calculator exam. If you are seen using a calculator, your exam will be collected and you will receive a zero on the exam and will be reported to the Honor Council.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero on the exam and will be reported to the Honor Council.
3. In Part 1 (Problems 1-8), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 5 points.
4. In Part 2 (Problems 11-14), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the **quality** and **correctness** of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________

DO NOT WRITE BELOW!

Question Type	Points Awarded	Points
Multiple Choice		40
Free Response		60
Total		100

PART I: Multiple Choice. 5 points each.

Part I Multiple choice.

1. Change $\int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} y\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$ to an equivalent integral in spherical coordinates.

$-2 \le x \le 0$

$0 \le y \le \sqrt{4-x^2}$

$0 \le z \le \sqrt{4-x^2-y^2}$

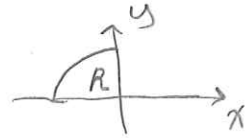
(a) $\int_{0}^{\pi/2}\int_{0}^{\pi/2}\int_{0}^{2} \rho^3 \sin^3\phi\, d\rho\, d\phi\, d\theta$

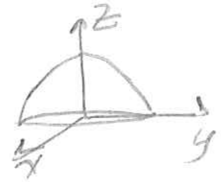
(b) $\int_{\pi/2}^{\pi}\int_{0}^{\pi/2}\int_{0}^{2} \rho^4 \sin^2\phi \sin\theta\, d\rho\, d\phi\, d\theta$ correct

(c) $\int_{\pi/2}^{\pi}\int_{0}^{\pi}\int_{0}^{2} \rho^4 \sin^2\phi \sin\theta\, d\rho\, d\phi\, d\theta$

(d) $\int_{\pi/2}^{\pi}\int_{0}^{\pi}\int_{0}^{2} \rho^3 \sin^2\phi \sin\theta\, d\rho\, d\phi\, d\theta$

(e) $\int_{0}^{\pi/2}\int_{0}^{\pi/2}\int_{0}^{2} \rho^4 \sin^3\phi \sin\theta\, d\rho\, d\phi\, d\theta$





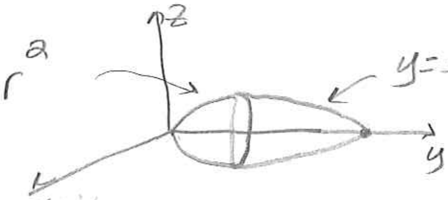
$\frac{\pi}{2} \le \theta \le \pi \qquad 0 \le \phi \le \frac{\pi}{2}$

$0 \le \rho \le 2$

$\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{\frac{\pi}{2}}\int_{0}^{2} \rho\sin\phi\sin\theta\, \rho\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

2. Which of the following is equivalent to $\iiint_E y\,dV$, where E is the solid bounded by the paraboloids $y = 2x^2 + 2z^2$ and $y = 36 - 2x^2 - 2z^2$?

$y = 2r^2$



(a) $\int_{0}^{2\pi}\int_{0}^{6}\int_{36-2r^2}^{2r^2} y\,dy\,r\,dr\,d\theta$

(b) $\int_{0}^{2\pi}\int_{0}^{3}\int_{36-2r^2}^{2r^2} r\sin\theta\,dy\,r\,dr\,d\theta$

(c) $\int_{0}^{2\pi}\int_{0}^{3}\int_{2r^2}^{36-2r^2} y\,dy\,r\,dr\,d\theta$ correct

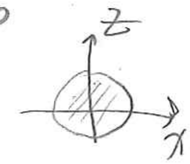
(d) $\int_{0}^{2\pi}\int_{0}^{6}\int_{2r^2}^{36-2r^2} y\,dy\,r\,dr\,d\theta$

(e) $\int_{0}^{2\pi}\int_{0}^{3}\int_{2r^2}^{36-2r^2} r\sin\theta\,dy\,r\,dr\,d\theta$

$2r^2 \le y \le 36-2r^2$ when paraboloids intersect $2x^2 + 2z^2 = 36 - 2x^2 - 2z^2$

$4x^2 + 4z^2 = 36$

$x^2 + z^2 = 9$



$0 \le r \le 3$

$0 \le \theta \le 2\pi$

$\int_{0}^{2\pi}\int_{0}^{3}\int_{2r^2}^{36-2r^2} y \cdot dy\, r\, dr\, d\theta$

3. Compute $\int_{-4}^{4}\int_{0}^{\sqrt{16-x^2}} \sin(x^2+y^2)\,dy\,dx$.

(a) $\frac{\pi}{2}(1-\cos 16)$ correct

(b) $\pi(\cos 16 - 1)$

(c) $-\frac{\pi}{2}(\cos 16)$

(d) $\frac{\pi}{2}(\cos 16)$

(e) $\frac{\pi}{2}(\cos 16 - 1)$

$0 \le y \le \sqrt{16-x^2}$

$-4 \le x \le 4$

$0 \le r \le 4$

$0 \le \theta \le \pi$

$$\int_0^{\pi}\int_0^4 \sin(r^2)\,r\,dr\,d\theta$$

$$\int_0^{\pi} -\frac{1}{2}\cos(r^2)\Big|_{r=0}^{r=4} d\theta$$

$$-\frac{1}{2}\int_0^{\pi}(\cos(16)-1) = \frac{-1}{2}\pi(\cos 16-1)$$

or $\frac{\pi}{2}(1-\cos 16)$

4. Find the volume of the solid below the cone $z=\sqrt{4x^2+4y^2}$ and above the ring $1 \le x^2+y^2 \le 9$, where the ring is in the xy-plane *to the right* of the y axis.

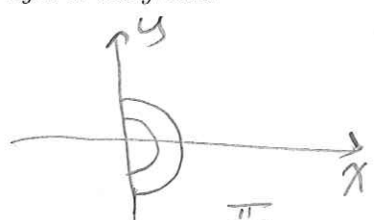
(a) 8π

(b) $\frac{104\pi}{3}$

(c) 18π

(d) $\frac{52\pi}{3}$ correct

(e) $\frac{26\pi}{3}$



$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$1 \le r \le 3$

$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_1^3 2r\cdot r\,dr\,d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_1^3 2r^2\,dr$$

$$= (\pi)\,\frac{2r^3}{3}\Big|_1^3$$

$$= \frac{2\pi}{3}(27-1) = \frac{52\pi}{3}$$

5. Which of the following integrals gives the area of the region bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$, in the *first quadrant*?

(a) $\displaystyle\int_0^{\pi/2} \int_2^{2\cos\theta} r\, dr\, d\theta$

(b) $\displaystyle\int_{\pi/4}^{\pi/2} \int_2^{2\cos\theta} r\, dr\, d\theta$

(c) $\displaystyle\int_0^{\pi/2} \int_{2\cos\theta}^{2} r\, dr\, d\theta$correct

(d) $\displaystyle\int_{\pi/4}^{\pi/2} \int_{2\cos\theta}^{2} r\, dr\, d\theta$

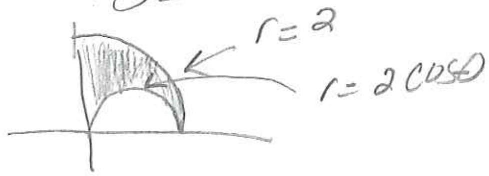
(e) None of the above

$r = 2$

$r = 2\cos\theta$

$2 = 2\cos\theta$

$\theta = 0$



$$A = \int_0^{\frac{\pi}{2}} \int_{2\cos\theta}^{2} r\, dr\, d\theta$$

6. Consider $\displaystyle\iint_R x^2 y\, dA$, where R is the parallelogram bounded by the lines $x - 3y = 0$, $3x - y = 8$, $3x - y = 7$ and $3x - y = 9$. If we use the transformation $u = x - 3y$ and $v = 3x - y$, which of the following is the absolute value of the Jacobian?

(a) 0

(b) $\frac{5}{32}$

(c) 10

(d) 8

(e) $\frac{1}{8}$ correct

$u = x - 3y$

$v = 3x - y$

$-3u = -3x + 9y$

$+\ v = 3x - y$

$-3u + v = 8y$

$y = \frac{1}{8}(-3u + v)$

$-3v = -9x + 3y$

$+\ u = x - 3y$

$u - 3v = -8x$

$x = -\frac{1}{8}(u - 3v)$

$$J = \begin{vmatrix} -\frac{1}{8} & \frac{3}{8} \\ -\frac{3}{8} & \frac{1}{8} \end{vmatrix} = -\frac{1}{64} + \frac{9}{64} = \frac{8}{64}$$

7. Change the rectangular point $(\sqrt{3}, -1, 2\sqrt{3})$ to cylindrical and spherical coordinates.

(a) Cylindrical: $\left(2, \frac{11\pi}{6}, 2\sqrt{3}\right)$; Spherical: $\left(4, \frac{11\pi}{6}, \frac{\pi}{6}\right)$ correct

(b) Cylindrical: $\left(2, \frac{2\pi}{3}, 2\sqrt{3}\right)$; Spherical: $\left(4, \frac{2\pi}{3}, \frac{\pi}{6}\right)$

(c) Cylindrical: $\left(2, \frac{11\pi}{6}, 2\sqrt{3}\right)$; Spherical: $\left(4, \frac{11\pi}{6}, \frac{\pi}{3}\right)$

(d) Cylindrical: $\left(2, \frac{2\pi}{3}, 2\sqrt{3}\right)$; Spherical: $\left(4, \frac{2\pi}{3}, \frac{\pi}{3}\right)$

(e) None of these

note: Q IV for θ.

$x = \sqrt{3}, \quad y = -1, \quad z = 2\sqrt{3}$

$r = 2 \quad \tan\theta = -\frac{1}{\sqrt{3}} \quad \theta = \frac{11\pi}{6}$

$\rho = 4$

$z = \rho\cos\phi \quad \phi = \frac{\pi}{6}$

8. Evaluate $\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{\sqrt{x^2+y^2}}^{4} xz\,dz\,dx\,dy$.

(a) $\frac{27}{2}$

(b) 0 correct

(c) 27

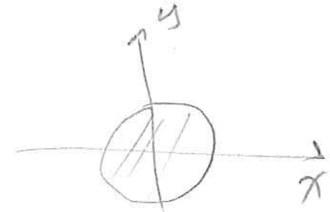
(d) 9

(e) $\frac{27}{4}$

$-3 \le y \le 3$

$-\sqrt{9-y^2} \le x \le \sqrt{9-y^2}$

$\sqrt{x^2+y^2} \le z \le 4$



$0 \le r \le 3$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^3\int_r^4 r\cos\theta\, z\, dz\, r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^3 r^2\cos\theta \left.\frac{z^2}{2}\right|_{z=r}^{z=4} dr\,d\theta$$

$$\int_0^{2\pi}\int_0^3 r^2\cos\theta\left(8 - \frac{r^2}{2}\right)dr\,d\theta$$

$$\int_0^{2\pi}\int_0^3 \cos\theta\left(8r^2 - \frac{r^4}{2}\right)dr\,d\theta$$

$$\rightarrow \underbrace{\int_0^{2\pi}\cos\theta\,d\theta}_{\downarrow\ 0}\int_0^3\left(8r^2 - \frac{r^4}{2}\right)dr$$

Part II: Work out. Show all intermediate steps. **Special note: For each integral, you must have its corresponding differential** (ie dx, dy ,dz, $d\theta$, $d\phi$, $d\rho$).

9. (10 pts) Set up but **do not evaluate** a triple integral that gives the volume of the solid E bounded by $y = x^2$ and the planes $z = 0$ and $y + z = 4$:

$z = 4 - y$

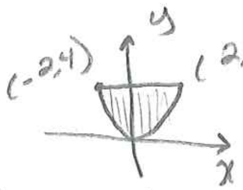
symmetry :

a.) In the order $dz\,dy\,dx$.

$0 \le z \le 4-y$

$y = x^2$

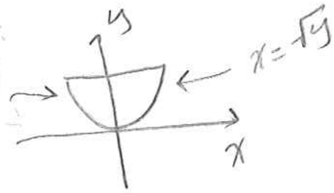
$y = 4$



$$V = \int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{4-y} dz\,dy\,dx$$

$x^2 \le y \le 4, \quad -2 \le x \le 2$

b.) In the order $dz\,dx\,dy$.

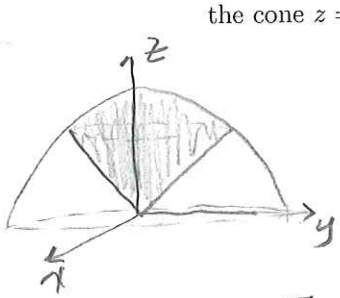


$$\int_{0}^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{4-y} dz\,dx\,dy$$

$-\sqrt{y} \le x \le \sqrt{y}$

$0 \le y \le 4$

10. (10 pts) Evaluate $\iiint_E z\,dV$, where E is the region bounded above by the sphere $x^2 + y^2 + z^2 = 9$ and below by the cone $z = \sqrt{3x^2 + 3y^2}$. Simplify your answer.



$\boxed{0 \le \rho \le 3}$

$\boxed{0 \le \theta \le 2\pi}$

$\boxed{0 \le \phi \le \frac{\pi}{6}}$

To find ϕ:

$z^2 = 3x^2 + 3y^2$

$\frac{z^2}{3} = x^2 + y^2$

$\frac{z^2}{3} + z^2 = 9$

$\frac{4z^2}{3} = 9$

$z^2 = \frac{27}{4}$

$z = \frac{3\sqrt{3}}{2}$

$z = \rho\cos\phi$

$\frac{3\sqrt{3}}{2} = 3\cos\phi$

$\phi = \frac{\pi}{6}$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{6}}\int_0^{3} \rho\cos\phi\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$\int_0^{2\pi} d\theta \int_0^{\frac{\pi}{6}} \cos\phi\sin\phi\, d\phi \int_0^{3} \rho^3\, d\rho$$

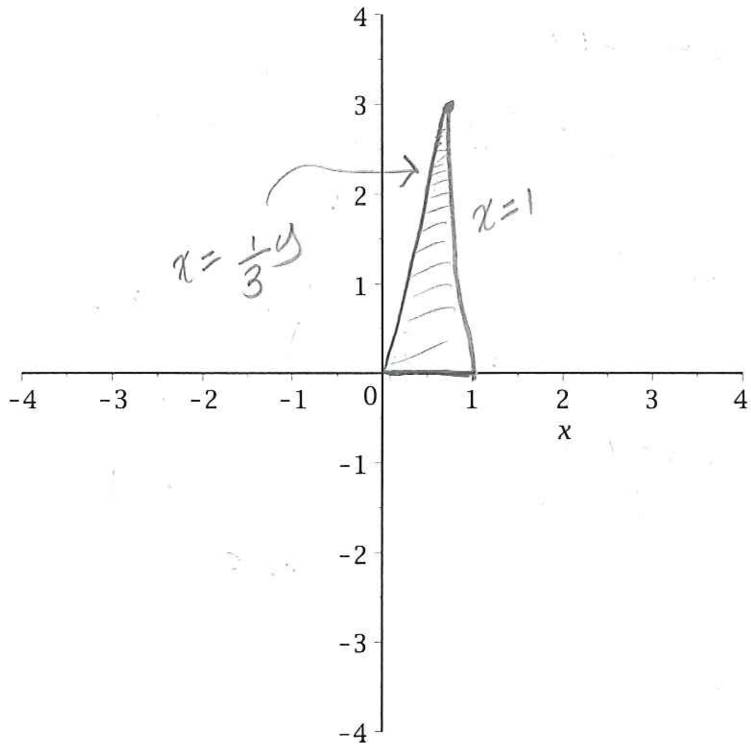
$$\theta\Big|_0^{2\pi}\ \frac{\sin^2\phi}{2}\Big|_0^{\frac{\pi}{6}}\ \frac{\rho^4}{4}\Big|_0^{3}$$

$$(2\pi)\left(\frac{1}{2}\left(\frac{1}{2}\right)^2\right)\frac{81}{4} = (2\pi)\left(\frac{1}{8}\right)\left(\frac{81}{4}\right)$$

$$= \boxed{\frac{81\pi}{16}}$$

11. Consider $\int_0^3 \int_{y/3}^1 \cos(x^2)\,dx\,dy$

a.) (5 pts) Sketch region of integration of integration in the xy-plane.



$\frac{y}{3} \le x \le 1$

$0 \le y \le 3$

$x = \frac{y}{3}, \quad x=1$

$y = 3x$

b.) (8 pts) Evaluate the integral.

$0 \le y \le 3x$

$0 \le x \le 1$

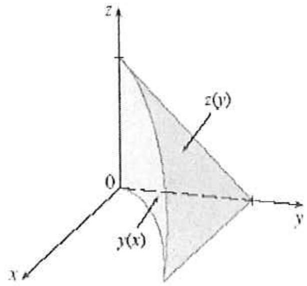
$$\int_0^1 \int_0^{3x} \cos(x^2)\,dy\,dx$$

$$\int_0^1 \cos(x^2)\, y \Big|_{y=0}^{y=3x} dx$$

$$\int_0^1 3x\cos(x^2)\,dx = \frac{3}{2}\sin(x^2)\Big|_0^1$$

$$= \boxed{\frac{3}{2}\sin(1)}$$

12. (10 points) Rewrite the integral $\int_0^{25}\int_{\sqrt{x}}^{5}\int_0^{5-y} f(x,y,z)\,dz\,dy\,dx$

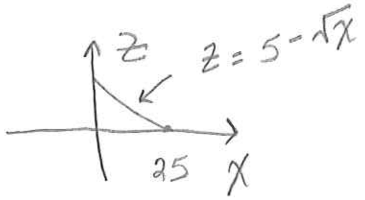


$0 \le z \le 5-y$

$\sqrt{x} \le y \le 5$

$0 \le x \le 25$

a.) In the order $dy\,dz\,dx$.



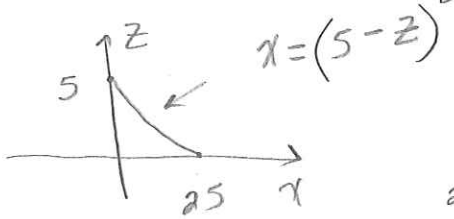
$z = 5-\sqrt{x}$ $\qquad$ $\sqrt{x} \le y \le 5-z$

$$\int_0^{25}\int_0^{5-\sqrt{x}}\int_{\sqrt{x}}^{5-z} f(x,y,z)\,dz\,dy\,dx$$

$0 \le z \le 5-\sqrt{x}$

$0 \le x \le 25$

b.) In the order $dy\,dx\,dz$.

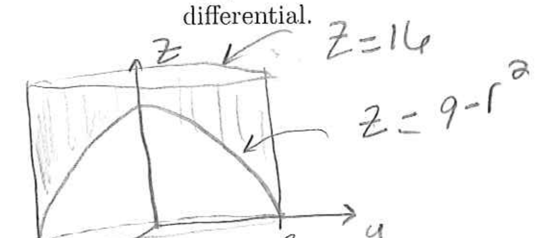


$$\int_0^{5}\int_0^{(5-z)^2}\int_{\sqrt{x}}^{5-z} f(x,y,z)\,dy\,dx\,dz$$

$0 \le x \le (5-z)^2$

$0 \le z \le 5$

13. (8 pts) Consider $\iiint_E xz\,dV$, where E is the region bounded above by the plane $z = 16$, below by the paraboloid $z = 9 - x^2 - y^2$, and inside the cylinder $x^2 + y^2 = 9$. Set up but **do not evaluate** an equivalent triple integral in cylindrical coordinates. Note: Be sure your limits of integration are clearly defined along with the appropriate differential.



$9-r^2 \le z \le 16$

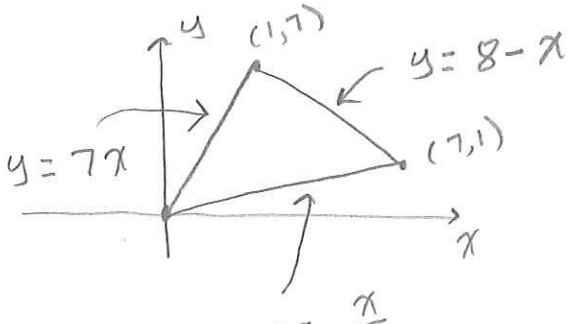
$0 \le \theta \le 2\pi$

$0 \le r \le 3$

$$\int_0^{2\pi}\int_0^{3}\int_{9-r^2}^{16} r\cos\theta\, z\,dz\,r\,dr\,d\theta$$

$r\cos\theta\, z\,dz\,dr\,d\theta$

14. (9 pts) Use the transformation $x = 7u + v$, $y = u + 7v$ to rewrite $\iint_R (x - 2y)\, dA$, where R is the triangluar region with vertices $(0, 0)$, $(7, 1)$, and $(1, 7)$ into an integral over a region S in the uv-plane. **Do not evaluate the integral.** Note: Be sure your limits of integration are clearly defined along with the appropriate differential.



$$J = \begin{vmatrix} 7 & 1 \\ 1 & 7 \end{vmatrix} = 48$$

$y = 7x$: $u + 7v = 7(7u + v)$

$u + 7v = 49u + 7v$

$\boxed{u = 0}$

$y = \frac{x}{7}$: $u + 7v = \frac{7u + v}{7}$

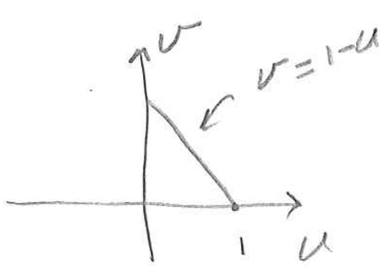
$7u + 49v = 7u + v$

$\boxed{v = 0}$

$y = 8 - x$: $u + 7v = 8 - 7u - v$

$8v + 8u = 8$

$\boxed{u + v = 1}$



$0 \le v \le 1 - u$

$0 \le u \le 1$

$$\int_0^1 \int_0^{1-u} \left[7u + v - 2(u + 7v)\right] 48 \, dv\, du$$

$$48 \int_0^1 \int_0^{1-u} (5u - 13v)\, dv\, du$$