

# Math 152 Week-in-Review

## Exam 2 Review

1. Find  $\int \frac{x^2}{\sqrt{9-x^2}} dx$ .

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$x = 3 \sin \theta$$

$$dx = 3 \cos \theta d\theta$$

$$\int \frac{(3 \sin \theta)^2}{\sqrt{9 - (3 \sin \theta)^2}} 3 \cos \theta d\theta$$

$$\sqrt{9 - 9 \sin^2 \theta}$$

$$\sqrt{9(1 - \sin^2 \theta)}$$

$$\sqrt{9 \cos^2 \theta}$$

$$3 \cos \theta$$

$$\int \frac{9 \sin^2 \theta}{3 \cos \theta d\theta} 3 \cos \theta d\theta$$

$$\int 9 \sin^2 \theta d\theta$$

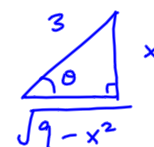
$$9 \int \frac{1}{2} [1 - \cos 2\theta] d\theta = \frac{9}{2} \left[ \theta - \frac{1}{2} \sin 2\theta \right]$$

$$\frac{9}{2} \left[ \theta - \frac{1}{2} \cdot 2 \sin \theta \cos \theta \right]$$

$$\boxed{\frac{9}{2} \left[ \arcsin\left(\frac{x}{3}\right) - \frac{x}{3} \cdot \frac{\sqrt{9-x^2}}{3} \right] + C}$$

$$\arcsin\left(\frac{x}{3}\right) = \theta$$

$$\frac{x}{3} = \sin \theta = \frac{\text{opp.}}{\text{hyp.}}$$



$$\sin \theta = \frac{x}{3}$$

$$\cos \theta = \frac{\sqrt{9-x^2}}{3}$$

2. Evaluate  $\int_0^{2/3} \frac{1}{(4+9x^2)^{5/2}} dx$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$x = \frac{2}{3} \tan \theta$$

$$dx = \frac{2}{3} \sec^2 \theta d\theta$$

Bounds:

$$\text{Top: } x = \frac{2}{3} = \frac{2}{3} \tan \theta$$

$$1 = \tan \theta$$

$$1 = \frac{\sin \theta}{\cos \theta} \quad \theta = \frac{\pi}{4}$$

$$\text{Bottom: } x = 0 = \frac{2}{3} \tan \theta$$

$$0 = \tan \theta$$

$$0 = \frac{\sin \theta}{\cos \theta} \quad \theta = 0$$

$$\int_0^{\pi/4} \frac{1}{(4+9(\frac{2}{3}\tan\theta)^2)^{5/2}} \cdot \frac{2}{3} \sec^2 \theta d\theta$$

$$[4+9 \cdot \frac{4}{9} \tan^2 \theta]^{5/2}$$

$$[4+4 \tan^2 \theta]^{5/2}$$

$$[4(1+\tan^2 \theta)]^{5/2}$$

$$[4 \sec^2 \theta]^{5/2}$$

$$[2 \sec \theta]^5$$

$$2^5 \sec^5 \theta$$

$$\int_0^{\pi/4} \frac{\frac{2}{3} \sec^2 \theta d\theta}{2^5 \sec^5 \theta}$$

$$\int_0^{\pi/4} \frac{\frac{1}{3} d\theta}{2^4 \sec^3 \theta} = \frac{1}{48} \int_0^{\pi/4} \frac{d\theta}{\sec^3 \theta}$$

$$\frac{1}{48} \int_0^{\pi/4} \cos^3 \theta d\theta$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

Bounds

$$\text{Top: } u(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$$

$$\text{Bottom: } u(0) = \sin(0) = 0$$

$$\frac{1}{48} \int_0^{\pi/4} \cos^2 \theta \cdot \cos \theta d\theta$$

$$\frac{1}{48} \int_0^{\pi/4} (1 - \sin^2 \theta) \cos \theta d\theta$$

$$\frac{1}{48} \int_0^{\sqrt{2}/2} (1-u^2) du = \frac{1}{48} \left[ u - \frac{u^3}{3} \right]_0^{\sqrt{2}/2}$$

$$\frac{1}{48} \left[ \frac{\sqrt{2}}{2} - \frac{(\frac{\sqrt{2}}{2})^3}{3} \right] = 0$$

Trig Sub Indicator:  $x^2 \pm a^2$  or  $a^2 \pm x^2$

3. After choosing the appropriate trigonometric substitution for  $\int (x-4)\sqrt{x^2-8x+7} dx$ , write the resulting integral in terms of  $\theta$ . Do NOT integrate.

$$(x \pm a)^2 = x^2 \pm 2ax + a^2$$

$$\int (x-4) \sqrt{(x-4)^2 - 9} dx$$

$$(x^2 - 8x + 16) + 7 - 16$$

$$(x-4)^2 - 9$$

$$x-4 = 3 \sec \theta$$

$$dx = 3 \sec \theta \tan \theta d\theta$$

$$(-2x^2 - 4x + 6)$$

$$-2(x^2 + 2x + 1) + 6 + 2$$

$$\int 3 \sec \theta \sqrt{(3 \sec \theta)^2 - 9} 3 \sec \theta \tan \theta d\theta$$

$$\sqrt{9 \sec^2 \theta - 9}$$

$$\sqrt{9(\sec^2 \theta - 1)}$$

$$\sqrt{9 \tan^2 \theta}$$

$$\int 3 \sec \theta 3 \tan \theta 3 \sec \theta \tan \theta d\theta$$

$$3 \tan \theta$$

$$\int 27 \sec^2 \theta \tan^2 \theta d\theta$$

or

$$\int 27 \frac{\sin^2 \theta}{\cos^4 \theta} d\theta$$

## Techniques of Integration

1. Antiderivative
5. u-sub
6. Integration by parts
2. Trig Integrals
4. Trig Substitution  $x^2 \pm a^2$  or  $a^2 \pm x^2$
3. Partial Fractions

4. Evaluate  $\int \frac{x^4 + 2x^3 - 6x^2 - 2x + 12}{x^3 + 4x^2 + 4x} dx$

$$= \int x - 2 + \frac{-2x^2 + 6x + 12}{x^3 + 4x^2 + 4x} dx$$

$$= \int x - 2 + \frac{-2x^2 + 6x + 12}{x(x+2)^2} dx$$

$$\left[ \frac{-2x^2 + 6x + 12}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \right] \times (x+2)^2 \quad \left( \frac{Dx+E}{x^2+9} \right)$$

$$-2x^2 + 6x + 12 = A(x+2)^2 + Bx(x+2) + Cx$$

Plug in

$x = -2$  :  $-2(-2)^2 + 6(-2) + 12 = A(0) + B(0) + C(-2)$   
 $-8 = -2C \quad C = 4$

$x = 0$  :  $12 = A(2)^2 + B(0) + C(0)$   
 $12 = 4A \quad A = 3$

$x = 1$  :  $-2 + 6 + 12 = 3(1+2)^2 + B(1)(1+2) + 4(1)$   
 $16 = 27 + 3B + 4$   
 $12 = 27 + 3B$   
 $-15 = 3B$   
 $B = -5$

$$\int x - 2 + \frac{3}{x} + \frac{-5}{x+2} + \frac{4}{(x+2)^2} dx$$

$$\boxed{\frac{x^2}{2} - 2x + 3 \ln|x| - 5 \ln|x+2| - \frac{4}{x+2} + C}$$

$$\left( \int \frac{-5}{2x+1} dx = \frac{-5}{2} \ln|2x+1| \right)$$

5. Evaluate  $\int_0^2 \frac{x^2 + 3x}{(x+1)(x^2+4)} dx$ .

$$\left[ \frac{x^2 + 3x}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \right] (x+1)(x^2+4)$$

$$x^2 + 3x = A(x^2+4) + (Bx+C)(x+1)$$

$$x = -1$$

$$1 + 3(-1) = A(5) + (Bx+C)(0)$$

$$-2 = 5A$$

$$A = -\frac{2}{5}$$

$$\underline{x^2 + 3x} = \underline{Ax^2} + \underline{4A} + \underline{Bx^2} + \underline{Cx} + \underline{Bx} + \underline{C}$$

Examine the coefficients:

$$x^2: \quad 1 = A + B$$

$$x: \quad 3 = B + C$$

$$1: \quad 0 = 4A + C$$

$$A = -\frac{2}{5}$$

$$1 = -\frac{2}{5} + B \quad B = \frac{7}{5}$$

$$0 = 4\left(-\frac{2}{5}\right) + C \quad C = \frac{8}{5}$$

$$\int \frac{-\frac{2}{5}}{x+1} + \frac{\frac{7}{5}x + \frac{8}{5}}{x^2+4} dx$$

$$-\frac{2}{5} \ln|x+1| + \frac{7}{5} \int \frac{x}{x^2+4} dx + \frac{8}{5} \int \frac{1}{x^2+4} dx$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right)$$

$$-\frac{2}{5} \ln|x+1| + \frac{7}{5} \cdot \frac{1}{2} \ln|x^2+4| + \frac{8}{5} \cdot \frac{1}{2} \arctan\left(\frac{x}{2}\right)$$

6. Evaluate  $\int_0^4 \frac{2}{(x-3)^2} dx$  or show it diverges.

$$\underbrace{\int_0^3 \frac{2}{(x-3)^2} dx}_{\text{Diverges}} + \int_3^4 \frac{2}{(x-3)^2} dx$$

$$\lim_{t \rightarrow 3^-} \int_0^t \frac{2}{(x-3)^2} dx$$

$$\lim_{t \rightarrow 3^-} \left. \frac{-2}{x-3} \right|_0^t$$

$$\lim_{t \rightarrow 3^-} \frac{-2}{t-3} + \frac{2}{0-3}$$

$$\frac{-2}{-0} = \infty$$

$$\boxed{\int_0^4 \frac{2}{(x-3)^2} dx \text{ Diverges}}$$

$$-1 \leq \sin(6x) \leq 1$$

7. Determine whether the improper integral converges or diverges using the comparison theorem.

a.)  $\int_1^{\infty} \frac{\sin(6x) + 8}{\sqrt{x} + x^4} dx \leq \int_1^{\infty} \frac{1 + 8}{x^4} dx$   $p$ -integral  $p=4 > 1$  Converge

$\int_1^{\infty} \frac{\sin(6x) + 8}{\sqrt{x} + x^4} dx$  converges by comparison theorem



$$1 \leq e^{1/x} \leq e$$

$$\text{b.) } \int_1^{\infty} \frac{e^{1/x} + 2}{x} dx \geq \int_1^{\infty} \frac{1 + 2}{x^1} dx \quad \begin{array}{l} \text{Diverges} \\ p\text{-integral } p=1 \end{array}$$

$$\int_1^{\infty} \frac{e^{1/x} + 2}{x} dx \text{ diverges by comparison theorem}$$

8. Find a general formula for the sequence  $\left\{ \frac{2}{1}, -\frac{7}{3}, \frac{12}{9}, -\frac{17}{27}, \frac{22}{81}, \dots \right\}$ . Assume the pattern continues, and the sequence begins with  $n = 1$ .

$$a_n = \left\{ \frac{(-1)^{n+1} (5n-3)}{3^{n-1}} \right\}$$

9. Consider the recursive sequence  $a_1 = 4$  and  $a_{n+1} = \frac{5}{6 - a_n}$ .

(a) Find the first three terms of the sequence.

$$\begin{aligned} a_1 &= 4 \\ a_2 &= \frac{5}{6-4} = \frac{5}{2} \\ a_3 &= \frac{5}{6-\frac{5}{2}} = \frac{5}{\frac{7}{2}} = \frac{10}{7} \\ a_4 &= \frac{5}{6-\frac{10}{7}} \end{aligned}$$

(b) Assuming the sequence is decreasing and bounded, determine if the sequence converges or diverges. If it converges, find the limit.

limit exists!

$$\lim_{n \rightarrow \infty} a_n = L$$

$$\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} \frac{5}{6 - a_n}$$

$$L = \frac{5}{6 - L}$$

$$(6 - L)L = 5$$

$$6L - L^2 = 5$$

$$0 = L^2 - 6L + 5$$

$$0 = (L - 5)(L - 1)$$

$$\cancel{L = 5}, \boxed{L = 1}$$

10. Determine whether the sequence converges or diverges. If it converges, what value does it converge to?

(a)  $a_n = \sin n$

Diverges by oscillation



$$(b) a_n = \frac{\sin n}{n}$$

$$\lim_{n \rightarrow \infty} \frac{-1}{n} \leq \lim_{n \rightarrow \infty} \frac{\sin n}{n} \leq \lim_{n \rightarrow \infty} \frac{1}{n}$$

$$0 \leq \lim_{n \rightarrow \infty} \frac{\sin n}{n} \leq 0$$

$$\lim_{n \rightarrow \infty} \frac{\sin n}{n} = 0$$

Converges,  $L=0$

$$(c) a_n = \cos\left(\frac{5}{n}\right)$$

$$\lim_{n \rightarrow \infty} \cos\left(\frac{5}{n}\right) = \cos\left(\lim_{n \rightarrow \infty} \frac{5}{n}\right) = \cos(0) = 1$$

Converges,  $L=1$

$$(d) a_n = \frac{(-1)^n n}{2n^2 + 1} \quad \text{Alternating!}$$

$$|a_n| = \frac{n}{2n^2 + 1}$$

$$\lim_{n \rightarrow \infty} \frac{n}{2n^2 + 1} = 0$$

$$\lim_{n \rightarrow \infty} \frac{(-1)^n n}{2n^2 + 1} = 0$$

Converges,  $L=0$

$$(e) \ a_n = \frac{(-1)^{n-1} n}{2+9n}$$

Alternating!

$$|a_n| = \frac{n}{2+9n}$$

$$\lim_{n \rightarrow \infty} \frac{n}{2+9n} = \frac{1}{9}$$

Diverges (by oscillation)



11. Determine if the following sequences are increasing, decreasing, or not monotonic.  
Also, determine if each sequence is bounded.

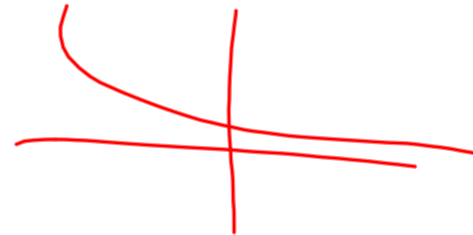
(a)  $a_n = 3 - e^{-2n}$

Let  $f(x) = 3 - e^{-2x}$

$f'(x) = 2e^{-2x} > 0$  (always positive)

$f(x)$  is increasing.

$a_n$  is also increasing



$\lim_{n \rightarrow \infty} 3 - e^{-2n} = 3 - e^{-\infty} = 3 - 0 = 3$  Converges  $\Rightarrow$  Bounded

(b)  $a_n = (-1)^n n$

$$a_1 = -1$$

$$a_2 = 2$$

$$a_3 = -3$$

$$a_4 = 4$$

Not monotonic

Not Bounded

$$\lim_{n \rightarrow \infty} n = \infty$$

12. For the series  $\sum_{n=1}^{\infty} a_n$ , the  $n$ th partial sum is given by  $s_n = \frac{3-2n}{5n+1}$ .

(a) Find  $a_5 = \underline{s_5} - \underline{s_4}$

$$s_5 = a_1 + a_2 + a_3 + a_4 + a_5$$

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$a_5 = \frac{3-2(5)}{5(5)+1} - \frac{3-2(4)}{5(4)+1}$$

$$= \boxed{\frac{-7}{26} - \frac{-5}{21}}$$

(b) Find  $\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} \frac{3-2n}{5n+1} = \boxed{-\frac{2}{5}}$

Series converges  $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

(c) What is  $\lim_{n \rightarrow \infty} a_n?$   $= 0$

13. Determine if the following series converges or diverges. If the series converges, find its sum.

(a)  $\sum_{n=1}^{\infty} \left[ \frac{1}{2^n} - \frac{1}{2^{n+1}} \right]$

$$S_n = \left( \frac{1}{2^1} - \cancel{\frac{1}{2^2}}^{a_1} \right) + \left( \cancel{\frac{1}{2^2}} - \cancel{\frac{1}{2^3}}^{a_2} \right) + \left( \cancel{\frac{1}{2^3}} - \cancel{\frac{1}{2^4}}^{a_3} \right) + \dots$$

$$\dots + \left( \cancel{\frac{1}{2^{n-2}}} - \cancel{\frac{1}{2^{n-1}}}^{a_{n-2}} \right) + \left( \cancel{\frac{1}{2^{n-1}}} - \cancel{\frac{1}{2^n}}^{a_{n-1}} \right) + \left( \cancel{\frac{1}{2^n}} - \frac{1}{2^{n+1}} \right)^{a_n}$$

$$S_n = \frac{1}{2} - \frac{1}{2^{n+1}}$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{1}{2} - \frac{1}{2^{n+1}} = \frac{1}{2} - 0 = \frac{1}{2}$$

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{2^n} - \frac{1}{2^{n+1}} = \frac{1}{2}} \quad \text{Converges}$$

$$(b) \sum_{n=1}^{\infty} \frac{6}{n(n+2)}$$

$$\left[ \frac{6}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} \right]_{n(n+2)}$$

$$6 = A(n+2) + Bn$$

$$n=-2: \quad 6 = -2B \quad B = -3$$

$$n=0: \quad 6 = 2A \quad A = 3$$

$$\sum_{n=1}^{\infty} \frac{3}{n} - \frac{3}{n+2}$$

$$S_n = \left( \frac{3}{1} - \frac{3}{3} \right) + \left( \frac{3}{2} - \frac{3}{4} \right) + \left( \frac{3}{3} - \frac{3}{5} \right) + \dots$$

$$\dots + \left( \frac{3}{n-2} - \frac{3}{n} \right) + \left( \frac{3}{n-1} - \frac{3}{n+1} \right) + \left( \frac{3}{n} - \frac{3}{n+2} \right)$$

$$S_n = 3 + \frac{3}{2} - \frac{3}{n+1} - \frac{3}{n+2}$$

$$\lim_{n \rightarrow \infty} 3 + \frac{3}{2} - \frac{3}{n+1} - \frac{3}{n+2} = 3 + \frac{3}{2} = \frac{9}{2}$$

$$\sum_{n=1}^{\infty} \frac{6}{n(n+2)} \text{ Converges, } S = \frac{9}{2}$$

Geometric

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{5 \cdot 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2 \cdot 2^n}{5 \cdot 3^n} = \sum_{n=1}^{\infty} \frac{2}{5} \cdot \frac{(-1)^n 2^n}{3^n}$$

$$\sum_{n=1}^{\infty} \frac{2}{5} \cdot \left( \frac{-2}{3} \right)^n$$

$$r = \frac{-2}{3}$$

$$|r| = \frac{2}{3} \Rightarrow \text{Converges}$$

$$|r| < 1 \Rightarrow \text{Converges}$$

$$|r| \geq 1 \Rightarrow \text{Diverges}$$

$$\text{Initial Term: } (n=1) \quad \frac{2}{5} \left( \frac{-2}{3} \right)^1 = \frac{-4}{15} = a$$

$$S = \frac{a}{1-r} = \frac{-4/15}{1 - \frac{-2}{3}} = \frac{-4/15}{5/3} = \frac{-4}{15} \cdot \frac{3}{5} = \frac{-12}{75} = \boxed{\frac{-4}{25}}$$

$$\boxed{\text{Converges, } S = \frac{-4}{25}}$$

Determine converges / diverges

$$(d) \sum_{n=1}^{\infty} \cos\left(\frac{1}{n^3}\right)$$

$$\sum_{n=1}^{\infty} a_n$$

$$a_n = \cos\left(\frac{1}{n^3}\right)$$

$$\lim_{n \rightarrow \infty} \cos\left(\frac{1}{n^3}\right) = \cos(0) = 1 \neq 0$$

$\Rightarrow \sum_{n=1}^{\infty} \cos\left(\frac{1}{n^3}\right)$  Diverges by Test for Divergence

14. Determine whether the following series converge or diverge. Support your answer.

$$(a) \sum_{n=1}^{\infty} \frac{n}{n+1}$$

$$a_n = \frac{n}{n+1}$$

$$\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1 \neq 0$$

$$\sum_{n=1}^{\infty} \frac{n}{n+1} \text{ Diverges by Test for Divergence}$$



Converge / Diverge

$$(b) \sum_{n=3}^{\infty} \frac{5}{n\sqrt{\ln n}}$$

$$f(x) = \frac{5}{x\sqrt{\ln x}}$$

cont. ?  $\checkmark$   
pos. ?  $\checkmark$   
dec. ?  $\checkmark$

$$\lim_{n \rightarrow \infty} \frac{5}{n\sqrt{\ln n}} = 0$$

$$\lim_{t \rightarrow \infty} \int_3^t \frac{5}{x\sqrt{\ln x}} dx$$

$$u = \ln x$$
$$du = \frac{1}{x} dx$$

$$\int \frac{5}{\sqrt{u}} du = \int 5u^{-1/2} du = 2 \cdot 5 u^{1/2}$$

$$\lim_{t \rightarrow \infty} 10 (\ln x)^{1/2} \Big|_3^t = \lim_{t \rightarrow \infty} 10 (\ln t)^{1/2} - 10 (\ln 3)^{1/2}$$

$\infty \Rightarrow$  Integral Diverges

$\Rightarrow \sum_{n=3}^{\infty} \frac{5}{n\sqrt{\ln n}}$  diverges by Integral Test

15. Using the remainder for the integral test, find an upper bound for the remainder

if we use  $s_8$  to approximate  $\sum_{n=1}^{\infty} \frac{1}{n^5}$  Converges, p-series  $p=5$

Remainder using 8 terms

(p-series is a shortcut for the integral test)

$$R_8 \leq \int_8^{\infty} f(x) dx$$

$$\text{where } f(x) = \frac{1}{x^5}$$

$$R_8 \leq \lim_{t \rightarrow \infty} \int_8^t \frac{1}{x^5} dx$$

$$\lim_{t \rightarrow \infty} \left. \frac{x^{-4}}{-4} \right|_8^t = \lim_{t \rightarrow \infty} -\frac{1}{4t^4} + \frac{1}{4 \cdot (8)^4} = \boxed{\frac{1}{4 \cdot (8)^4}} = \frac{1}{16384}$$

16. Using the remainder for the integral test, what is the smallest value of  $n$  that ensures  $s_n$  to approximate  $\sum_{n=1}^{\infty} \frac{3}{n^4}$  with error less than  $\frac{1}{100}$ ?

↑  
Converges by p-series  $p=4$

We want:  $R_n \leq \frac{1}{100}$

We know  $R_n \leq \int_n^{\infty} \frac{3}{x^4} dx \leq \frac{1}{100}$

...

$$\frac{1}{n^3} \leq \frac{1}{100}$$

$$100 \leq n^3$$

$$n \geq \sqrt[3]{100} \approx 4.6$$

$$\boxed{n=5}$$

17. Determine if the following statements are true or false. If the statement is false, give a counter example

(a) If a sequence converges, then it is bounded.

True

(b) If a sequence is bounded, then it converges.

False,  $a_n = \sin(n)$

(c) If a sequence is increasing, then it converges.

False,  $a_n = n$

(d) If  $\lim_{n \rightarrow \infty} s_n = 4$ , then  $\sum_{n=1}^{\infty} a_n$  converges.

True  $\sum_{n=1}^{\infty} a_n = 4$

(e) If  $\sum_{n=1}^{\infty} a_n$  converges, then  $\lim_{n \rightarrow \infty} a_n = 0$ .

True

(f) The p-series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges if  $p \leq 1$ . should say  $p > 1$ .

False,  $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$  Diverges

(g) The geometric series  $\sum_{n=2}^{\infty} ar^{n-1}$  converges if  $|r| < 1$ .

True.