Spring 2019 Math 152

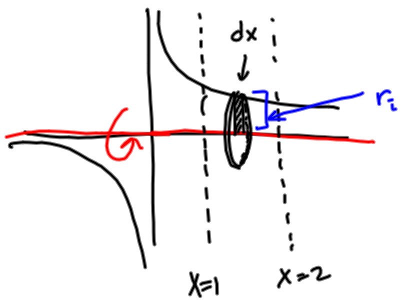
Week in Review 2

courtesy: Amy Austin

(covering sections 6.2)

Section 6.2

1. Find the volume of the solid obtained by revolving the region bounded by $y = \dfrac{1}{x}$, $y = 0$, $x = 1$, $x = 2$ about the x-axis.



A_i = Area of a slice $= \pi (r_i)^2$

$r_i = \frac{1}{x} - 0$

$A_i = \pi\left(\frac{1}{x}\right)^2$ $\quad V_i = \pi\left(\frac{1}{x^2}\right) dx$

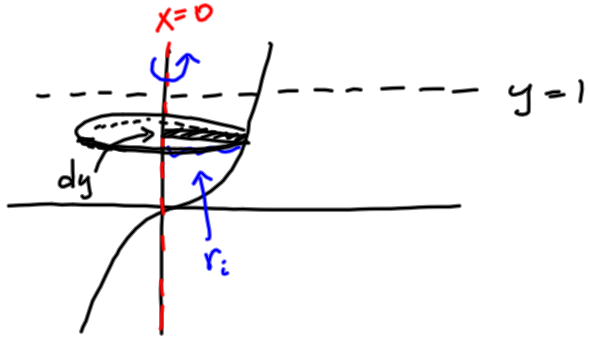
$V = \int V_i = \int_1^2 \pi \frac{1}{x^2}\, dx$

$\int_a^b \pi [f(x)]^2\, dx$

$\pi \int_1^2 x^{-2}\, dx = \pi\left(-x^{-1}\right)\Big|_1^2$

$\pi\left[-\frac{1}{2} - -1\right] = \boxed{\frac{\pi}{2}}$

2. Find the volume of the solid obtained by revolving the region bounded by $y = 3x^5$, $y = 1$ and $x = 0$ about the y-axis.



$r_i = \left(\frac{y}{3}\right)^{1/5} - 0$

$y = 3x^5$

$\frac{y}{3} = x^5$

$A_i = \pi \left(\frac{y}{3}\right)^{2/5}$

$x = \left(\frac{y}{3}\right)^{1/5}$

$V = \pi \int_0^1 \left(\frac{y}{3}\right)^{2/5} dy$

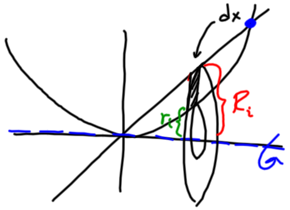
$\left(\frac{y}{3}\right)^{2/5} = \frac{y^{2/5}}{3^{2/5}}$

$= \frac{\pi}{3^{2/5}} \int_0^1 y^{2/5} \, dy$

$= \frac{\pi}{3^{2/5}} \cdot \frac{5}{7} \cdot y^{7/5} \Big|_0^1 = \boxed{\frac{5\pi}{3^{2/5} \cdot 7}}$

3. Find the volume of the solid obtained by revolving the region bounded by $y = x^2$ and $y = 4x$ about the x-axis, then the y axis.

(a) x-axis



$R_i = 4x - 0$

$r_i = x^2 - 0$

$x^2 = 4x$

$x^2 - 4x = 0$

$x\ (x-4) = 0$

$x = 0 \qquad x - 4 = 0$

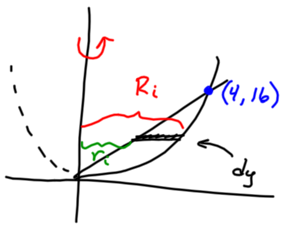
$x = 4$

$A_i = \pi(4x)^2 - \pi(x^2)^2$

$$V = \pi \int_0^4 16x^2 - x^4 \, dx$$

$$= \pi \left[\frac{16}{3}x^3 - \frac{1}{5}x^5\right]_0^4 = \boxed{\pi \cdot \left[\frac{16}{3}(4^3) - \frac{1}{5}(4^5)\right]}$$

(b) y-axis



$y = 4x \qquad y = x^2$

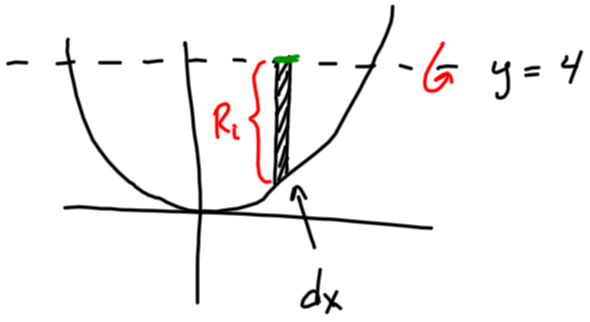
$x = \frac{y}{4} \qquad x = y^{1/2}$

$R_i = y^{1/2} - 0$

$r_i = \frac{y}{4} - 0$

$$V = \pi \int_0^{16} (y^{1/2})^2 - \left(\frac{y}{4}\right)^2 dy$$

4. Find the volume of the solid obtained by revolving the region bounded by $y = x^2$, $y = 4$, about the line $y = 4$.



$R_i = 4 - x^2$

$$\boxed{V = \pi \int_{-2}^{2} (4-x^2)^2 \, dx \quad \text{or} \quad = 2\pi \int_{0}^{2} (4-x^2)^2 \, dx}$$

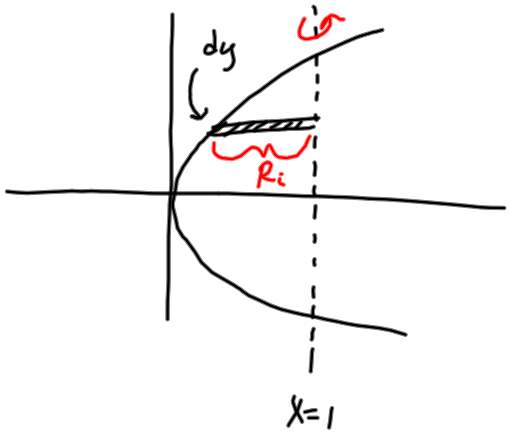
$x^2 = 4$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2 \quad x = 2$

5. Find the volume of the solid obtained by revolving the region bounded by $x = y^2$, $x = 1$, about the line $x = 1$.



$R_i = 1 - y^2$

$$\boxed{\begin{gathered} V = \pi \int_{-1}^{1} (1-y^2)^2 \, dy \\ \text{or} \\ V = 2\pi \int_{0}^{1} (1-y^2)^2 \, dy \end{gathered}}$$

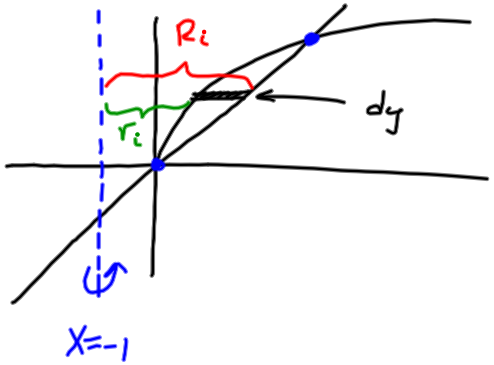
$1 = y^2$

$y^2 - 1 = 0$

$(y+1)(y-1) = 0$

$y = -1 \quad y = 1$

6. Find the volume of the solid obtained by revolving the region bounded by $y = x$, $y = \sqrt{x}$, about the line $x = -1$.



$y = x$ $\qquad$ $y = \sqrt{x}$

$x = y$ $\qquad$ $x = y^2$

$y = y^2$

$y^2 - y = 0$

$y(y-1) = 0$

$y = 0 \qquad y = 1$

$R_i = y - (-1) = y + 1$

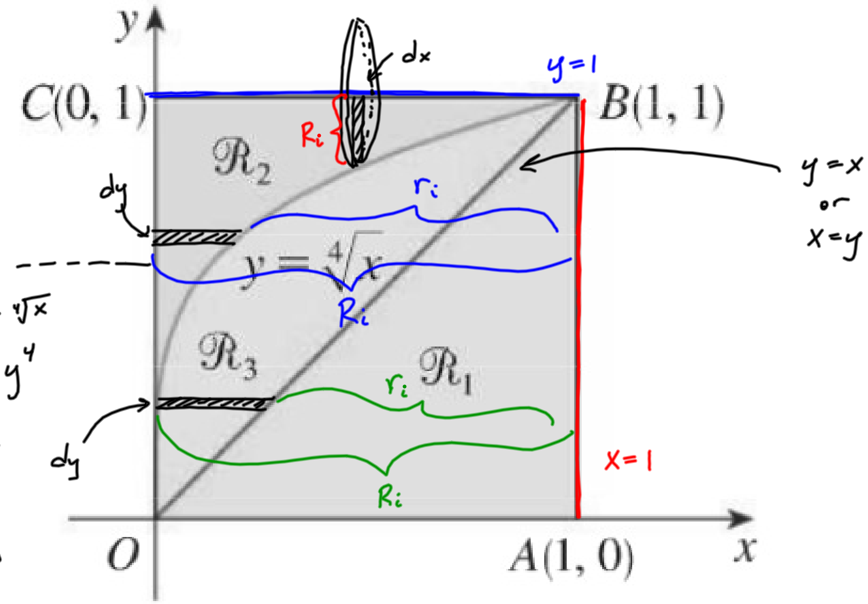
$r_i = y^2 - (-1) = y^2 + 1$

$$\boxed{V = \pi \int_0^1 (y+1)^2 - (y^2+1)^2 \, dy}$$

7. Refer to the figure below to set up but do not evaluate an integral that finds the volume generated by rotating the given region about the specified line.

(a) $R_i = 1 - \sqrt[4]{x}$

$$\boxed{V = \pi \int_0^1 (1 - \sqrt[4]{x})^2 \, dx}$$



(b) $R_i = 1 - y^4$

$r_i = 1 - y$

$y = \sqrt[4]{x}$

$x = y^4$

$$\boxed{V = \pi \int_0^1 (1-y^4)^2 - (1-y)^2 \, dy}$$

(a) R_2 about BC

(b) R_3 about AB

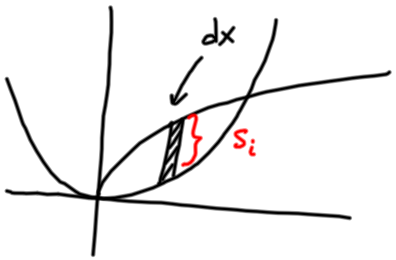
(c) R_2 about AB

(c) $R_i = 1 - 0$

$r_i = 1 - y^4$

$$\boxed{V = \pi \int_0^1 1^2 - (1-y^4)^2 \, dy}$$

8. Find the volume of S where the base of S is the region bounded by $y = x^2$ and $y = \sqrt{x}$. The cross sections perpendicular to the x-axis are squares.



$A_i = s_i^2$

$V_i = s_i^2 \; dx$

$x^2 = \sqrt{x}$

$S_i = \sqrt{x} - x^2$

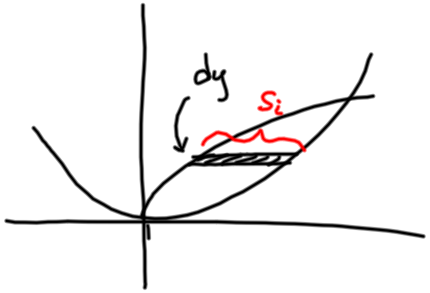
$A_i = (\sqrt{x} - x^2)^2$

$= x - 2\sqrt{x} \cdot x^2 + x^4$

$= x - 2x^{5/2} + x^4$

$$\boxed{V = \int_0^1 (x - 2x^{5/2} + x^4) \, dx}$$

9. Find the volume of S where the base of S is the region bounded by $y = x^2$ and $y = \sqrt{x}$. The cross sections perpendicular to the y-axis are squares.



$y = x^2$

$x = y^{1/2}$

$y = \sqrt{x}$

$x = y^2$

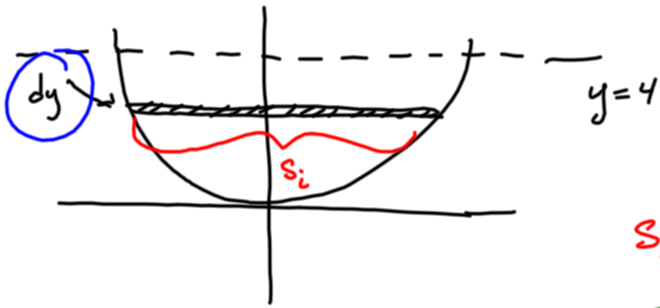
$y^{1/2} = y^2$

$S_i = y^{1/2} - y^2$

$A_i = \left(y^{1/2} - y^2\right)^2$

$$\boxed{V = \int_0^1 \left(y^{1/2} - y^2\right)^2 \, dy}$$

10. Find the volume of the solid S where the base of S is the region bounded by $y = x^2$ and $y = 4$. The cross-sections perpendicular to the y axis are equilateral triangles.

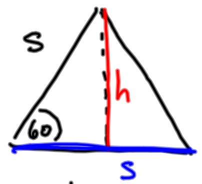


$y = x^2$

$x = \pm y^{1/2}$

$S_i = y^{1/2} - (-y^{1/2})$

$S_i = 2\, y^{1/2}$



$\sin(60) = \frac{h}{s}$

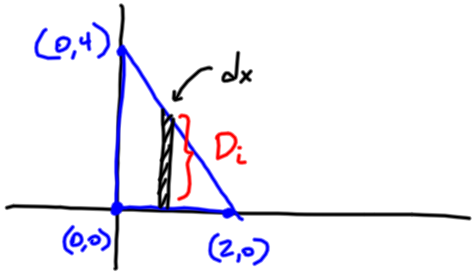
$h = s \cdot \sin(60) = s \cdot \frac{\sqrt{3}}{2}$

$A = \frac{1}{2} bh = \frac{1}{2}\, s\, s\, \frac{\sqrt{3}}{2}$

$A = \frac{s^2 \sqrt{3}}{4}$

$$\boxed{V = \int_0^4 \frac{(2\, y^{1/2})^2 \sqrt{3}}{4}\, dy}$$

11. Find the volume of the solid S where the base of S is the triangular region with vertices $(0,0)$, $(2,0)$ and $(0,4)$. Cross-sections perpendicular to the x axis are semi-circles.



$D_i = 2r_i$

$\frac{D_i}{2} = r_i$

$A_i = \frac{1}{2}\pi(r_i)^2$

$A_i = \frac{1}{2}\pi\left(\frac{D_i}{2}\right)^2 = \frac{\pi}{8}(D_i)^2$

$m = \frac{4-0}{0-2} = \frac{4}{-2} = -2$

$y = -2x + 4$

$D_i = (-2x+4) - 0$

$A_i = \frac{\pi}{8}(-2x+4)^2$

$$\boxed{V = \int_0^2 \frac{\pi}{8}(-2x+4)^2\,dx}$$

12. Find the volume of the solid S where the base of S is the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{16}=1$. Cross sections perpendicular to the x-axis are ~~isosceles triangles~~ where the base and height are equal.

$$\frac{x^2}{4}+\frac{y^2}{16}=1$$

$$\frac{y^2}{16}=1-\frac{x^2}{4}$$

$$y^2=16-4x^2$$

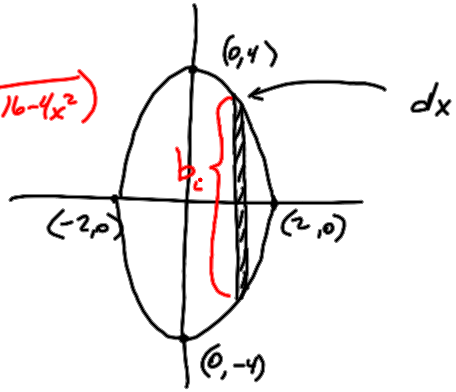
$$y=\pm\sqrt{16-4x^2}$$

$$b_i=\sqrt{16-4x^2}-\left(-\sqrt{16-4x^2}\right)$$

$$b_i=2\sqrt{16-4x^2}$$

$$h_i=b_i$$

$$A_i=\frac{1}{2}\,b_i\cdot h_i=\frac{1}{2}(b_i)^2$$



$$A_i=\frac{1}{2}\left(2\sqrt{16-4x^2}\right)^2=\frac{1}{2}\cdot 4\cdot(16-4x^2)=2(16-4x^2)$$

$$\boxed{V=\int_{-2}^{2}2(16-4x^2)\,dx}\quad \text{or}\quad \boxed{V=2\int_{0}^{2}2(16-4x^2)\,dx}$$