Fall 2019 Math 152

Week in Review 6

courtesy: Amy Austin

(covering section 11.4-11.5)

Section 11.4

1. Determine whether the following series converge or diverge.

a.) $\sum_{n=1}^{\infty} \frac{n^4}{10n^4 + n^2 + 1}$

$$\lim_{n\to\infty} \frac{n^4}{10n^4+n^2+1} = \frac{1}{10} \neq 0$$

Diverges by Test for Divergence

b.) $\sum_{n=1}^{\infty} \frac{n^2}{n^5 + 10n + 1} \leq \sum_{n=1}^{\infty} \frac{n^2}{n^5} = \sum_{n=1}^{\infty} \frac{1}{n^3}$

Converges,
p-series $p = 3 > 1$ ✓

$\sum_{n=1}^{\infty} \frac{n^2}{n^5 + 10n + 1}$ Converges, by Comparison Test c.) $\sum_{n=3}^{\infty} \frac{n^2+n+9}{\sqrt{n^5-n^2-1}} \geq \sum_{n=3}^{\infty} \frac{n^2}{\sqrt{n^5-n^2-1}} \geq \sum_{n=3}^{\infty} \frac{n^2}{\sqrt{n^5}}$

$$\sum_{n=3}^{\infty} \frac{n^2}{n^{5/2}} = \sum_{n=3}^{\infty} \frac{1}{n^{1/2}}$$

Diverges

p-series $p = 1/2 \not> 1$

$\sum_{n=3}^{\infty} \frac{n^2+n+9}{\sqrt{n^5-n^2-1}}$ Diverges by the Comparison Test d.) $\sum_{n=2}^{\infty} \frac{\cos^2 n + 5}{n^3 + \sqrt{n}} \le \sum_{n=2}^{\infty} \frac{\cos^2(n) + 5}{n^3} \le \sum_{n=2}^{\infty} \frac{1+5}{n^3}$ Converges p-series $p = 3 > 1$

$0 \le \cos^2(n) \le 1$

$\sum_{n=2}^{\infty} \frac{\cos^2 n + 5}{n^3 + \sqrt{n}}$ Converges, by Comparison Test e.) $\sum_{n=1}^{\infty} \frac{5+\sin(7n)}{\sqrt{n}}$ $\geq$ $\sum_{n=1}^{\infty} \frac{5-1}{n^{1/2}}$ Diverge

p-series $p = \frac{1}{2} \ngtr 1$

$-1 \leq \sin(7n) \leq 1$

$\sum_{n=1}^{\infty} \frac{5+\sin(7n)}{\sqrt{n}}$ Diverges by Comparison Test f.) $\sum_{n=1}^{\infty} \frac{5+\arctan(n)}{n^3}$ $\leq$ $\sum_{n=1}^{\infty} \frac{5+\frac{\pi}{2}}{n^3}$ Converge

p-series $p=3>1$

Range of arctan (n) is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\frac{-\pi}{2} \leq \arctan n \leq \frac{\pi}{2}$

$\sum_{n=1}^{\infty} \frac{5+\arctan n}{n^3}$ Converges, by Comparison Test g.) $\sum_{n=1}^{\infty} \frac{3}{4^n + n}$ $\leq$ $\sum_{n=1}^{\infty} \frac{3}{n}$ Diverges

p-series $p = 1 \ngtr 1$

No good

Not useful

$$\sum_{n=1}^{\infty} \frac{3}{4^n+n} \leq \sum_{n=1}^{\infty} \frac{3}{4^n} = \sum_{n=1}^{\infty} 3 \cdot \left(\frac{1}{4}\right)^n$$

Geometric Series!

$r = \frac{1}{4}$

$|r| = \frac{1}{4} < 1$

Converges

$\sum_{n=1}^{\infty} \frac{3}{4^n+n}$ Converges, by Comparison Test h.) $\sum_{n=5}^{\infty} \frac{n}{8n^2+6n+1} \leq \sum_{n=5}^{\infty} \frac{n}{8n^2} = \sum_{n=5}^{\infty} \frac{1}{8n}$ Diverges p-series $p=1$ $\not> 1$

Direct Comparison Test is not useful here.

Must use Limit Comparison Test instead.

$$\sum_{n=5}^{\infty} \frac{n}{8n^2+6n+1} \sim \sum_{n=5}^{\infty} \frac{n}{8n^2}$$

$$\lim_{n\to\infty} \frac{\frac{n}{8n^2+6n+1}}{\frac{n}{8n^2}} = \lim_{n\to\infty} \frac{\cancel{n}}{8n^2+6n+1} \cdot \frac{8n^2}{\cancel{n}} = 1 = L$$

$0 < L < \infty$ ✓ $\Rightarrow$ The two series have the same behavior.

$\sum_{n=5}^{\infty} \frac{n}{8n^2+6n+1}$ Diverges by Limit Comparison Test.

i.) $\sum_{n=1}^{\infty} \frac{n^4 - n^3 + 1}{\sqrt{n^{10} - n^6 + 3}}$ $\sim$ $\underbrace{\sum_{n=1}^{\infty} \frac{n^4}{\sqrt{n^{10}}}}$ $= \sum_{n=1}^{\infty} \frac{n^4}{n^5} = \underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}$ Diverges p-series $p = 1 \not> 1$

$$\lim_{n\to\infty} \frac{n^4 - n^3 + 1}{\sqrt{n^{10} - n^6 + 3}} \cdot \frac{\sqrt{n^{10}}}{n^4} = \lim_{n\to\infty} \left(\frac{n^4 - n^3 + 1}{n^4}\right) \cdot \left(\frac{\sqrt{n^{10}}}{\sqrt{n^{10} - n^6 + 3}}\right)$$

$$= 1 \cdot 1 = 1 = L$$

$0 < L < \infty$ ✓

$\sum_{n=1}^{\infty} \frac{n^4 - n^3 + 1}{\sqrt{n^{10} - n^6 + 3}}$ Diverges by L.C.T.

j.) $\sum_{n=1}^{\infty} \frac{n^2+5n-2}{(3n+1)^3+2n}$ $\sim \sum_{n=1}^{\infty} \frac{n^2}{(3n)^3} = \sum_{n=1}^{\infty} \frac{n^2}{27n^3} = \sum_{n=1}^{\infty} \frac{1}{27n}$

Diverges

p-series

$p = 1 \ngtr 1$

$$\lim_{n\to\infty} \frac{n^2+5n-2}{(3n+1)^3+2n} \cdot \frac{(3n)^3}{n^2} = \lim_{n\to\infty} \frac{3^3 \cdot n^5 + \cdots}{3^3 \cdot n^5 + \cdots} = 1 = L$$

$0 < L < \infty$ ✓

$\sum_{n=1}^{\infty} \frac{n^2+5n-2}{(3n+1)^3+2n}$ Diverges by L.C.T.

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^3}\right) \sim \sum_{n=1}^{\infty} \frac{1}{n^3}$$

Converges
p-series
$p = 3 > 1$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{n \to \infty} \frac{\sin(n^{-3})}{n^{-3}} \overset{L'H}{=} \lim_{n \to \infty} \frac{\cos(n^{-3}) \cdot -3n^{-4}}{-3n^{-4}}$$

$$= \lim_{n \to \infty} \cos\left(\frac{1}{n^3}\right) = \cos(0) = 1 = L$$

$0 < L < \infty$ ✓

$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^3}\right)$ Converges by L.C.T.

Title : Nov 6-7:58 PM (Page 12 of 17)

Section 11.5

4. Use the alternating series test to determine whether the following series converge.

a.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ Alternating!

(1) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$ ✓

$b_n = |a_n| = \frac{1}{\sqrt{n+1}}$

(2) b_n decreasing? ✓

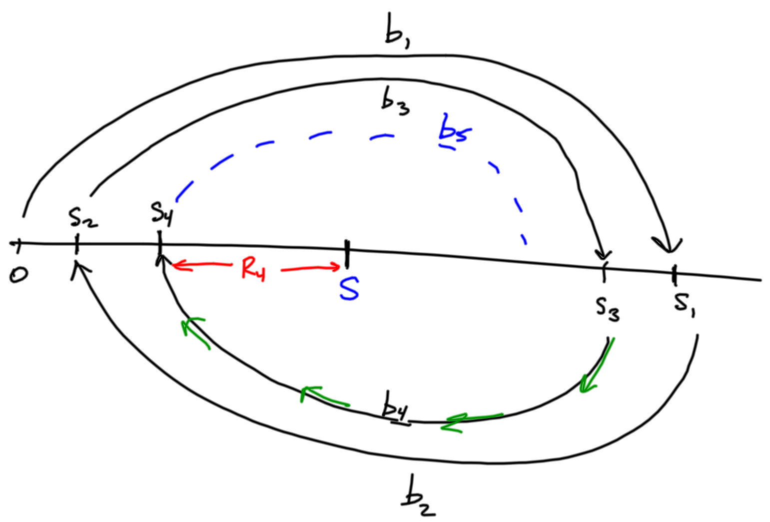
$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ Converges by Alternating Series Test

A.S.T.

$R_4 = |S_4 - S|$

$|R_4| \le b_5$

$|R_n| \le b_{n+1}$



b.) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+n^2}$ Alternating!

(1) $\lim_{n\to\infty} \frac{n^2}{1+n^2} = 1 \neq 0$ X

$b_n = \frac{n^2}{1+n^2}$

$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+n^2}$ Diverges by Test for Divergence c.) $\sum_{n=1}^{\infty} (-1)^{n-1} 2^{-n}$ Alternating

(1) $\lim_{n \to \infty} \frac{1}{2^n} = \frac{1}{\infty} = 0$ ✓

$b_n = 2^{-n} = \frac{1}{2^n}$

(2) b_n Decreasing? ✓

$\sum_{n=1}^{\infty} (-1)^{n-1} 2^{-n}$ Converges by A.S.T.

d.) $\sum_{n=1}^{\infty}(-1)^n 2^{3/n}$

Alternating !

(1) $\lim_{n\to\infty} 2^{3/n} = 2^{3/\infty} = 2^{0} = 1 \neq 0$ X

$b_n = 2^{3/n}$

$\boxed{\sum_{n=1}^{\infty}(-1)^n 2^{3/n} \text{ Diverges by Test for Divergence}}$

5. Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ Alternating!

$b_n = \frac{1}{n^2}$

a.) Prove the series is convergent.

(1) $\lim_{n\to\infty} \frac{1}{n^2} = 0$ ✓

(2) b_n Decreasing? ✓

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ Converges by A.S.T.

b.) Use s_6 to approximate the sum of the series and use the Alternating Series Estimation Theorem to estimate the error in using the 6th partial sum to approximate the sum of the series.

$$|R_6| \leq b_7 = \frac{1}{7^2} = \frac{1}{49}$$

$$|R_6| \leq \frac{1}{49}$$

c.) Determine the minimum number of terms we need to add in order to find the sum with error less than $\frac{1}{120}$.

Find n,

We want:

$$|R_n| < \frac{1}{120}$$

We know

$$|R_n| \leq b_{n+1} < \frac{1}{120}$$

$$b_{n+1} = \frac{1}{(n+1)^2} < \frac{1}{120}$$

$$\sqrt{120} < \sqrt{(n+1)^2}$$

$$10.9 < n+1$$

$$9.9 < n$$

$\boxed{n = 10 \text{ terms}}$