Section 5.5

1. $\int \frac{1+x^2-x}{\sqrt{x}}\,dx = \int\left(\frac{1}{\sqrt{x}} + \frac{x^2}{\sqrt{x}} - \frac{x}{\sqrt{x}}\right)dx$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$= \int\left(x^{-\frac{1}{2}} + x^{\frac{3}{2}} - x^{\frac{1}{2}}\right)dx$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \boxed{2x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C}$

2. $\int_0^1 (x^3-2)^2\,dx = \int_0^1 (x^3-2)(x^3-2)\,dx$

$= \int_0^1 \left(x^6 - 4x^3 + 4\right)dx$

$= \underbrace{\frac{x^7}{7} - \frac{\cancel{4}x^4}{\cancel{4}} + 4x}_{F(x)}\Big|_0^1$

$= F(1) - F(0)$

$= \frac{1}{7} - 1 + 4 - 0$

$= 3 + \frac{1}{7} = \boxed{\frac{22}{7}}$

$u = 3x^3 - 1$

$du = 9x^2\,dx$

$\frac{du}{9x^2} = dx$

3. $\int 5\cancel{x^2}(\underbrace{3x^3-1}_{u})^8 \underbrace{dx}_{\frac{du}{9\cancel{x^2}}} =$

$\frac{5}{9}\int u^8\,du$

$\frac{5}{9}\frac{u^9}{9} + C = \boxed{\frac{5}{81}\left(3x^3-1\right)^9 + C}$

4. $\displaystyle\int_{0=x}^{1=x} x^2 e^{2x^3-5}\,dx =$

$u = 2x^3 - 5$; $x=1,\ u=-3$; $x=0,\ u=-5$

$du = 6x^2\,dx$

$\dfrac{du}{6x^2} = dx$

$$\int_{-5}^{-3} \cancel{x^2}\, e^u \frac{du}{6\cancel{x^2}}$$

$$\frac{1}{6}\int_{-5}^{-3} e^u\,du = \frac{1}{6} e^u \Big|_{-5=u}^{-3=u}$$

$$= \frac{1}{6}\left(e^{-3} - e^{-5}\right) = \frac{1}{6}\left(\frac{1}{e^3} - \frac{1}{e^5}\right)$$

5. $\displaystyle\int_{-4}^{0} \frac{1}{\sqrt{1-2x}}\,dx = \int_{-4}^{0} (1-2x)^{-\frac{1}{2}}\,dx$

$u = 1-2x$; $x=0,\ u=1$; $x=-4,\ u=9$

$du = -2\,dx$

$\dfrac{du}{-2} = dx$

$$\int_9^1 u^{-\frac{1}{2}} \frac{du}{-2}$$

Recall from 151

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$-\frac{1}{2}\int_9^1 u^{-\frac{1}{2}}\,du$$

$$\frac{1}{2}\int_1^9 u^{-\frac{1}{2}}\,du = \cancel{\frac{1}{2}}\,\frac{u^{\frac{1}{2}}}{\cancel{\frac{1}{2}}}\Bigg|_1^9 = 9^{\frac{1}{2}} - 1^{\frac{1}{2}}$$

$$= \sqrt{9} - 1$$

$$= \boxed{2}$$

6. $\displaystyle\int_1^{1/2} \cos \pi x\,dx =$ $\quad \to -\displaystyle\int_{\frac{1}{2}}^{1} \cos(\pi x)\,dx$

$u = \pi x$; $x = \frac{1}{2},\ u = \frac{\pi}{2}$; $x=1,\ u=\pi$

$du = \pi\,dx$

$\dfrac{du}{\pi} = dx$

$$\frac{1}{\pi}\int_{\pi}^{\frac{\pi}{2}} \cos u\,du$$

$$-\int_{\frac{1}{2}}^{1} \cos \pi x\,dx \to -\frac{1}{\pi}$$

$$\frac{1}{\pi}\sin u\Big|_{\pi}^{\frac{\pi}{2}} = \frac{1}{\pi}\Big(\underbrace{\sin\tfrac{\pi}{2}}_{1} - \underbrace{\sin \pi}_{0}\Big)$$

$$= \boxed{\frac{1}{\pi}}$$

7. $\int_0^{\pi/12} \tan(3x)\,dx = \int_0^{\frac{\pi}{12}} \frac{\sin 3x}{\boxed{\cos 3x}}\,dx$

$u = \cos(3x)$ $x=\frac{\pi}{12}, u=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$; $x=0, u=\cos 0=1$

$du = \boxed{-3}\sin(3x)\,dx$

$-\frac{1}{3}\int_1^{\frac{\sqrt{2}}{2}} \frac{du}{u} = \frac{1}{3}\int_{\frac{\sqrt{2}}{2}}^{1} \frac{du}{u}$

$= \frac{1}{3}\ln|u| \Big|_{\frac{\sqrt{2}}{2}}^{1}$

$= \frac{1}{3}\left(\cancel{\ln(1)}^{0} - \ln\frac{\sqrt{2}}{2}\right)$

$= -\frac{1}{3}\ln\frac{\sqrt{2}}{2}$

web Assign # 8. $\int \frac{(\ln(5x))^{24}}{x}\,dx$ du

$= \int u^{24}\,du$

$= \frac{u^{25}}{25} + C = \boxed{\frac{(\ln(5x))^{25}}{25} + C}$

8. $\int \left(\frac{1}{\sqrt{1-x^2}} - 4x^{-1} + e^x + \frac{2}{x^2+1} - \frac{1}{x^2+4} \right) dx =$

$\boxed{\arcsin x - 4\ln|x| + e^x + 2\arctan x - \frac{1}{2}\arctan\frac{x}{2} + C}$

9. $\int \frac{e^x}{1+e^x}\,dx =$ du

$u = 1+e^x$

$du = e^x\,dx$

$\int \frac{du}{u} = \ln|u| + C$

$= \ln|1+e^x| + C$

$= \ln(1+e^x) + C$, since $1+e^x > 0$

10. $\int_1^2 \frac{5}{2x+1}\,dx =$

$u = 2x+1$ $x=2, u=5$; $x=1, u=3$

$du = 2dx$

$\frac{5}{2}\int_3^5 \frac{du}{u} = \frac{5}{2}\ln|u| \Big|_3^5$

$= \frac{5}{2}(\ln 5 - \ln 3) = \boxed{\frac{5}{2}\ln\frac{5}{3}}$

11. $\displaystyle\int \frac{\sin t}{\cos^5 t}\,dt =$

$\cancel{u = \cos^5 t}$

$\cancel{du = 5\cos^4 t(-\sin t)\,dt}$

$u = \cos t$

$du = -\sin t\,dt$

$$-\int \frac{du}{u^5} = -\int u^{-5}\,du$$

$$= -\frac{u^{-4}}{-4} + C$$

$$= \frac{1}{4u^4} + C$$

$$= \boxed{\frac{1}{4\cos^4 t} + C}$$

12. $\displaystyle\int \frac{x}{\sqrt{x+1}}\,dx =$

$u = x+1 \longrightarrow x = u-1$

$du = dx$

$$\int \frac{u-1}{\sqrt{u}}\,du = \int\left(\frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\right)du$$

$$= \int\left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$$

$$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \boxed{\frac{2}{3}(x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} + C}$$

13. $\displaystyle\int \frac{2x^3}{x^2-1}\,dx =$

$u = x^2 - 1 \longrightarrow u+1 = x^2$

$du = 2x\,dx$

$\dfrac{du}{2x} = dx$

$$\int \frac{\cancel{2}x^{\cancel{3}\,2}}{u}\,\frac{du}{\cancel{2x}}$$

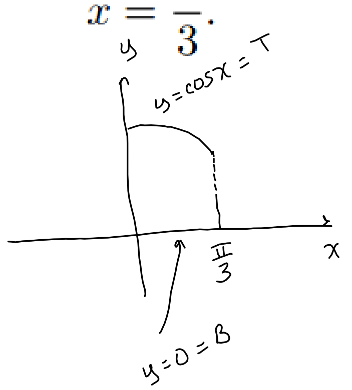
$$\int \frac{x^2}{u}\,du = \int \frac{u+1}{u}\,du = \int\left(1 + \frac{1}{u}\right)du$$

$$= u + \ln|u| + C$$

$$= \boxed{x^2 - 1 + \ln|x^2-1| + C}$$

Section 6.1

14. Find the area bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \frac{\pi}{3}$.



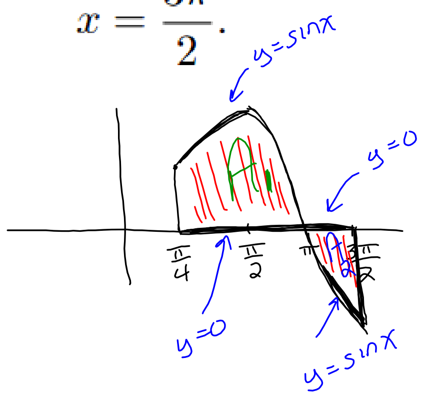
$$A = \int_a^b (T - B)\,dx$$

$$= \int_0^{\frac{\pi}{3}} (\cos x - 0)\,dx$$

$$= \sin x \Big|_0^{\frac{\pi}{3}}$$

$$= \frac{\sqrt{3}}{2} - 0$$

15. Find the area bounded by $y = \sin x$, $y = 0$, $x = \frac{\pi}{4}$, $x = \frac{3\pi}{2}$.



$$A = A_1 + A_2$$

$$= \int_{\frac{\pi}{4}}^{\pi} (\sin x - 0)\,dx + \int_{\pi}^{\frac{3\pi}{2}} (0 - \sin x)\,dx$$

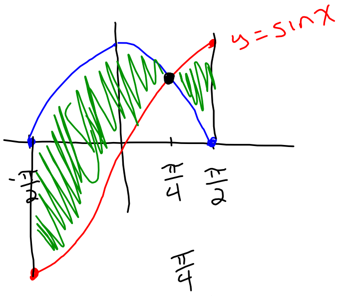
$$= -\cos x \Big|_{\frac{\pi}{4}}^{\pi} + \cos x \Big|_{\pi}^{\frac{3\pi}{2}}$$

$$= -\left(\cos\pi - \cos\frac{\pi}{4}\right) + \cos\frac{3\pi}{2} - \cos\pi$$

$$= -\left(-1 - \frac{\sqrt{2}}{2}\right) + 0 + 1$$

$$= \boxed{2 + \frac{\sqrt{2}}{2}}$$

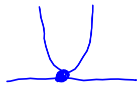
16. Find the area bounded by $y = \sin x$, $y = \cos x$, $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.



$\sin x = \cos x$

$x = \frac{\pi}{4}$

$$\int_{-\pi/2}^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$



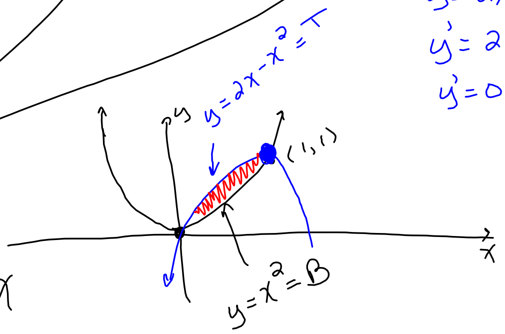
17. Find the area bounded by $y = x^2$ and $y = 2x - x^2$.

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$\boxed{x = 0,\ x = 1}$



$y = 2x - x^2$

$y' = 2 - 2x$

$y' = 0 \quad x = 1$

$(1,1)$

$$A = \int_0^1 (2x - x^2 - x^2)\,dx$$

$$= \int_0^1 (2x - 2x^2)\,dx$$

$$= x^2 - \frac{2x^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$$

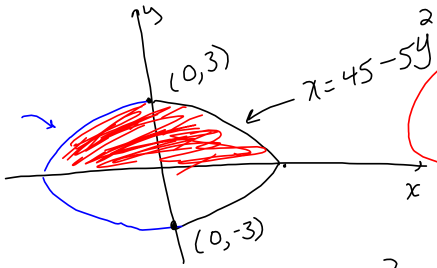
18. Find the area bounded by $x = 45 - 5y^2$ and $x = 5y^2 - 45$.

$45 - 5y^2 = 5y^2 - 45$

$90 = 10y^2$

$y = \pm 3$

$x = 0$



$A = \int_{-3}^{3} (R - L)\, dy = \int_{-3}^{3} 45 - 5y^2 - (5y^2 - 45)\, dy$

$= \int_{-3}^{3} (90 - 10y^2)\, dy = 360$

symmetry!

$2\int_0^3 45 - 5y^2 - (5y^2 - 45)\, dy$

$2\int_0^3 (90 - 10y^2)\, dy$

$2\left(90y - \frac{10y^3}{3}\right)\Big|_0^3$

$2(270 - 90)$

$2(180) = 360$

19. Sketch the region R bounded by $x = y^2$ and $x = 5y + 6$. Set up but do not evaluate an integral in terms of y and then an integral in terms of x that gives the area of this region.

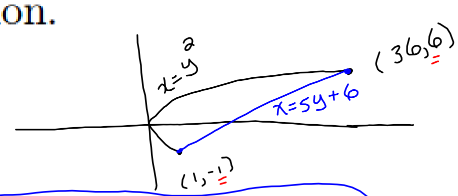
$y^2 = 5y + 6$

$y^2 - 5y - 6 = 0$

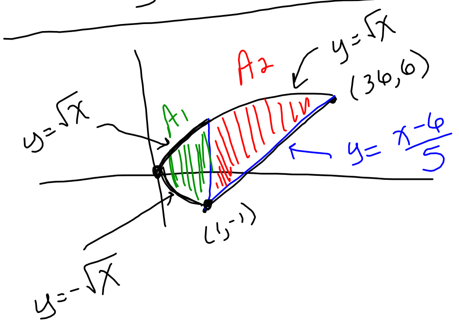
$(y - 6)(y + 1) = 0$

$y = 6, \quad x = 36$

$y = -1, \quad x = 1$



$$\boxed{\begin{aligned} A &= \int (R - L)\, dy \\ &= \int_{-1}^{6} (5y + 6 - y^2)\, dy \end{aligned}}$$



$x = y^2, \quad x = 5y + 6$

$y = \pm\sqrt{x}, \quad y = \frac{x - 6}{5}$

$A = A_1 + A_2$

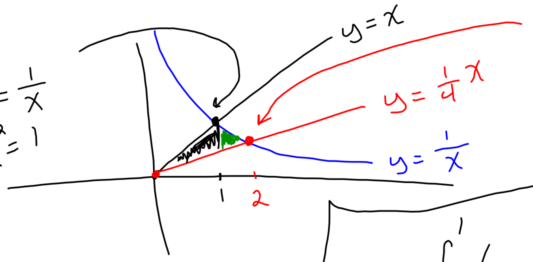
$= \int_0^1 \sqrt{x} - (-\sqrt{x})\, dx + \int_1^{36} \left(\sqrt{x} - \frac{x - 6}{5}\right) dx$

20. Sketch the region R bounded by $y = \frac{1}{x}$, $y = x$, $y = \frac{1}{4}x$, $x \geq 0$. Set up but do not evaluate an integral that gives the area of R.

$y = x, \quad y = \frac{1}{4}x$

$x = \frac{1}{x}$

$x^2 = 1$



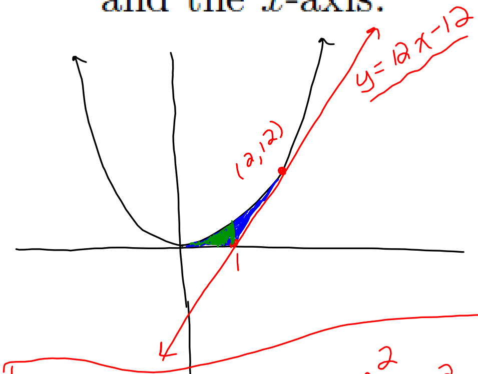
$\frac{1}{4}x = \frac{1}{x}$

$x^2 = 4$

$x = 2$

$$\boxed{A = \int_0^1 \left(x - \frac{1}{4}x\right)dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right)dx}$$

21. Find the area of the region bounded by the parabola $y = 3x^2$, the tangent line to this parabola at $(2, 12)$ and the x-axis.



Tangent line: $m = f'(2)$

$f(x) = 3x^2$

$f'(x) = 6x$

$m = 12$, point $(2,12)$

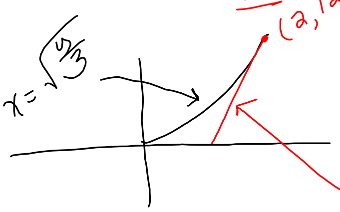
$y - y_1 = m(x - x_1)$

$y - 12 = 12(x - 2)$

$\boxed{y = 12x - 12}$

$$\boxed{A = \int_0^1 (3x^2 - 0)\,dx + \int_1^2 3x^2 - (12x - 12)\,dx}$$

or



$y = 3x^2$

$x = \sqrt{\frac{y}{3}}$

$y = 12x - 12$

$x = \frac{y+12}{12}$

$$\boxed{A = \int_0^{12} \left(\frac{y+12}{12} - \sqrt{\frac{y}{3}} \right) dy}$$