1. Find the volume of the solid obtained by rotating the region bounded by the given curve(s) about the specified axis.

a.) $y = 10x - x^2$, $y = 0$ about the y axis.

$y = x(10-x)$

x-int: $x=0$, $x=10$

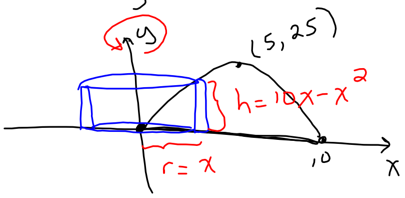
vertex: $f'(x)=0$

$10-2x=0$

$10=2x$

$x=5$

$y=25$

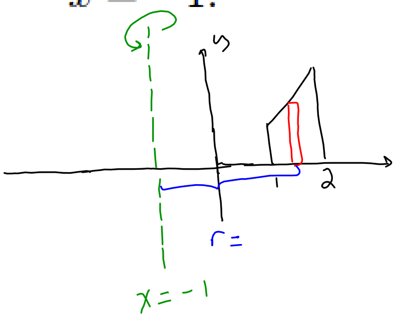


$$V = \int_0^{10} 2\pi\, x(10x-x^2)\,dx$$

$$= 2\pi \int_0^{10} (10x^2 - x^3)\,dx$$

$$= 2\pi \left(\frac{10x^3}{3} - \frac{x^4}{4}\right)\Big|_0^{10}$$

b.) $y = x^3$, $y = 0$, $x = 1$, $x = 2$, about the line $x = -1$.



$r =$ right $-$ left

$= x - (-1)$

$r = x+1$

$h = x^3$

$$V = \int_1^2 2\pi(x+1)x^3\,dx$$

$$= 2\pi \int_1^2 (x^4 + x^3)\,dx$$

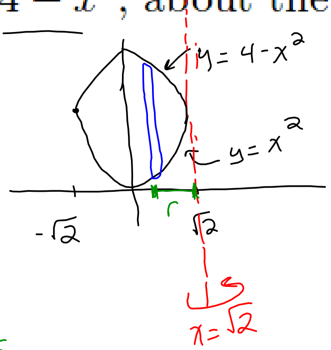
c.) $y = x^2$ and $y = 4 - x^2$, about the line $x = \sqrt{2}$.

$x^2 = 4 - x^2$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$



$r = \sqrt{2} - x$

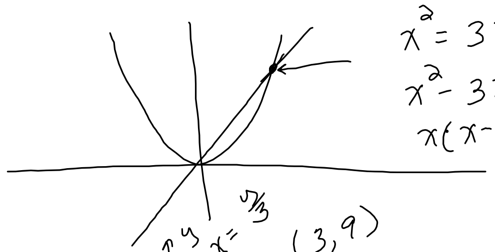
$h = T - B$

$= 4 - x^2 - (x^2)$

$= 4 - 2x^2$

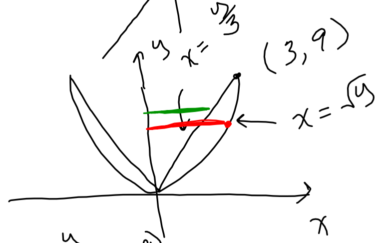
$$V = \int_{-\sqrt{2}}^{\sqrt{2}} 2\pi(\sqrt{2} - x)(4-2x^2)\,dx$$

2. Using two different methods, set up but do not evaluate the integral that gives the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 3x$, about the y axis.



$x^2 = 3x$
$x^2 - 3x = 0$
$x(x-3) = 0$

$x=0,\ x=3$
$y=0,\ y=9$

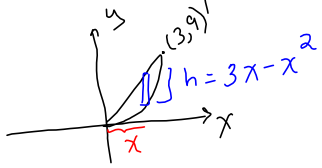


$R = \sqrt{y}$
$r = \frac{y}{3}$

washer: dy

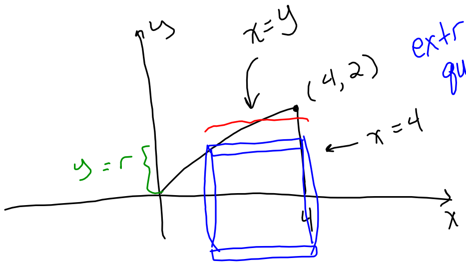
$V = \int_0^9 \pi (R^2 - r^2)\,dy$

$V = \pi \int_0^9 \left(y - \frac{y^2}{9}\right) dy$



shell: $V = \int_0^3 2\pi x (3x - x^2)\,dx$

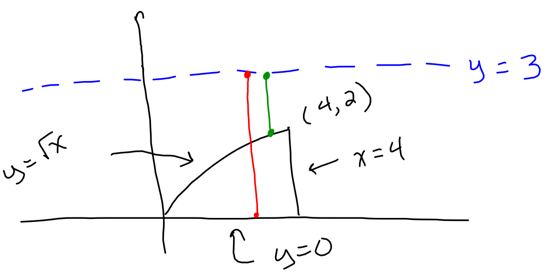
3. Using two different methods, set up but do not evaluate the integral that gives the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $x = 0$, $x = 4$, $y = 0$, about the line $y = 3$.



extra question

$\boxed{\begin{array}{l} \text{① rotate about } x\text{-axis using shells: } r = y \\ h = \text{right} - \text{left} = 4 - y^2 \\ V = \int_0^2 2\pi (y)(4 - y^2)\,dy \end{array}}$

② rotate about the line $y=3$



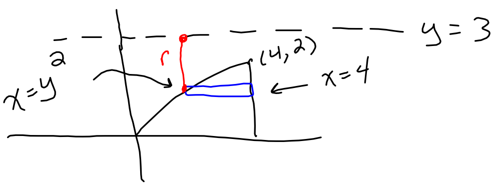
- washer:

$V = \int_0^4 \pi (R^2 - r^2)\,dx$

$R = 3 - 0 = 3$

$r = 3 - \sqrt{x}$

$V = \int_0^4 \pi \left(9 - (3 - \sqrt{x})^2\right) dx$



- shells

$r = 3 - y$

$h = 4 - y^2$

$V = \int_0^2 2\pi (3 - y)(4 - y^2)\,dy$

4. How much work is done in lifting a 30 lb barbell from the floor to a height of 4 feet?

$W = F \cdot d$ $F =$ "weight"

$d =$ distance moved

$W = (30 \text{ lbs})(4 \text{ feet})$

$\boxed{W = 120 \text{ ft-lbs}}$

5. When a particle is at a distance x meters from the origin, a force of $f(x) = 3x^2 + 2$ Newtons acts on it. How much work is done in moving the object from $x = 2$ to $x = 4$?

Force <u>not</u> constant, then $W = \int$ force

$W = \int_2^4 (3x^2 + 2)\, dx$

6. A spring has a natural length of 6 inches. If a 5-lb force is required to maintain it to a length of 18 inches, how much work is required to stretch it from 1 foot to 3 feet?

Hooke's Law: The force required to hold a spring stretched x units beyond its natural length is $f(x) = k x$

natural lenth = 6 inches.
maintain to 18 inches = 12 inches from natural length = 1 foot
requires 5 pounds.

$f(x) = kx$

$5 = k(1)$ $k = 5 \rightarrow \boxed{f(x) = 5x}$

work to stretch it from 1 foot to 3 feet

$$1 - \tfrac{1}{2} = \tfrac{1}{2} \qquad 3 - \tfrac{1}{2} = \tfrac{5}{2}$$

$$W = \int_{\frac{1}{2}}^{\frac{5}{2}} 5x\,dx$$

7. Suppose the work needed to stretch a spring from its natural length to a length of 5 feet beyond its natural length is 30 ft-lb.

a.) How much work is done in stretching the spring from 3 feet beyond its natural length to 120 inches beyond its natural length?

since 30-ft-lbs of work is done $\int_0^5 kx\,dx = 30$

$$\frac{kx^2}{2}\Big|_0^5 = 30$$

$$\frac{25}{2}k = 30$$

$$k = 30 \cdot \frac{2}{25}$$

$$\boxed{k = \frac{12}{5}}$$

$f(x) = \frac{12}{5}x$

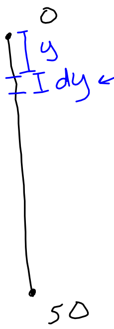
$$W = \int_3^{10} \frac{12}{5}x\,dx$$

b.) How far beyond its natural length will a force of 60 lb keep the spring stretched?

$f(x) = \frac{12}{5}x$ solve $f(x) = 60$ for x.

$$60 = \frac{12}{5}x \rightarrow x = 60\left(\frac{5}{12}\right) \text{ ft}$$

8. A heavy rope, 50 feet long, weighs 0.5 pounds per foot and hangs over the edge of a building 120 feet high. How much work is done in pulling the rope to the top of the building?



weighs $0.5\,dy$ lbs

moved y feet

$$W = 0.5y\,dy$$

$$W_{total} = \int_0^{50} .5y\,dy$$

force = total weight

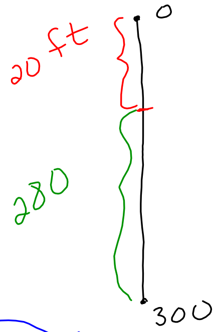
$$f(x) = \text{Total weight} - \tfrac{1}{2}x$$

$$= (50)(0.5) - \tfrac{1}{2}x$$

$$W = \int_0^{50} \left(25 - \tfrac{1}{2}x\right)dx$$

9. A 200 pound cable is 300 feet long and hangs vertically from the top of a tall building. How much work is required to pull 20 feet of the cable to the top of the building?

rope weighs $\frac{200 \text{ lbs}}{300 \text{ ft}} = \frac{2}{3}\frac{\text{lbs}}{\text{foot}}$



$$W = \int_0^{20} \tfrac{2}{3}x\,dx + \underline{(280 \text{ ft})\left(\tfrac{2}{3}\tfrac{\text{lbs}}{\text{ft}}\right)(20 \text{ ft})}$$

\#

$$= \tfrac{2}{3}\tfrac{x^2}{2}\Big|_0^{20} + \underline{\ \#\ }$$

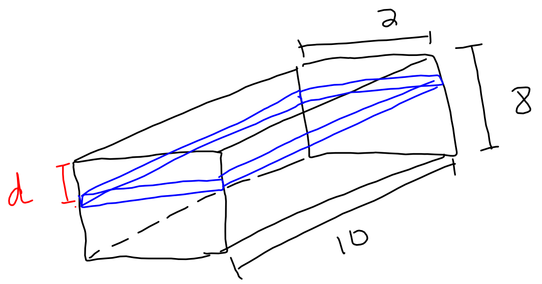
method 2

$$f(x) = \text{Total weight} - \tfrac{2}{3}x$$

$$f(x) = 200 - \tfrac{2}{3}x$$

$$W = \int_0^{20}\left(200 - \tfrac{2}{3}x\right)dx$$

10. An aquarium 10 m long, 2 m wide and 8 m deep is full of water. Find the work required to pump the top 3 feet of water to the top of the aquarium.



(a) Find work done in pumping all the water to the top.

ρg = weight density

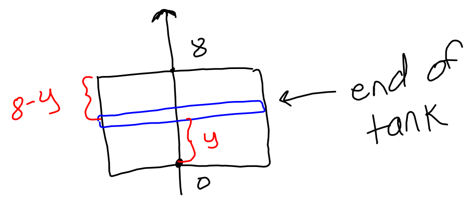
$\rho g = 9800 \frac{N}{m^3}$

$V_{slice} = (2m)(10m)\,dy\,m$

$V_{slice} = 20\,dy$

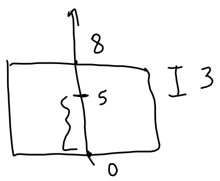
$F_{slice} = (20)(9800)\,dy$

$W_{slice} = (20)(9800)(8-y)\,dy$



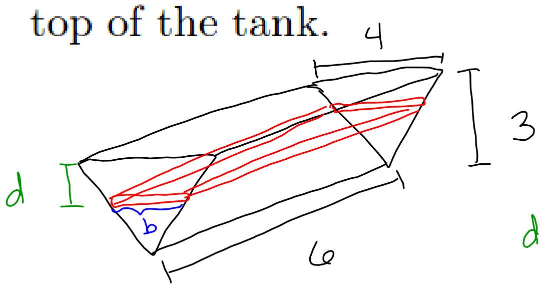
(a) empty entire tank, $W = \int_0^8 (20)(9800)(8-y)\,dy$

(b) only pump top 3 feet to the top.

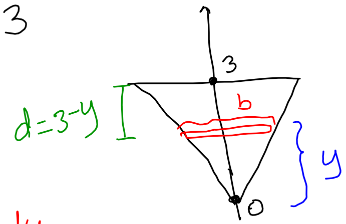


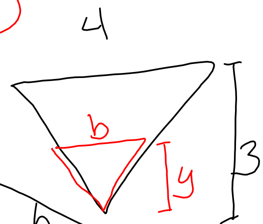
$W = \int_5^8 (20)(9800)(8-y)\,dy$

11. A tank contains water and has the shape of a trough 6 feet long. The end of the trough is an isosceles triangle with height 3 feet and base length 4 feet. The vertex of the triangle is at the bottom. Find the work required to pump all of the water to the top of the tank.



$V_{slice} = (b)(6)(dy)$





$\frac{4}{3} = \frac{b}{y}$

$b = \frac{4}{3}y$

$V_{slice} = \frac{4}{3} y (6)\, dy$

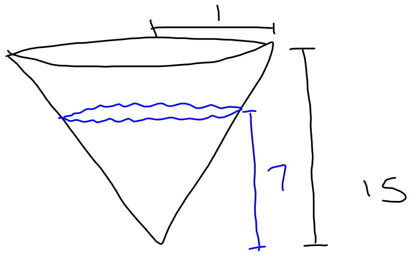
$V_{slice} = 8y\, dy$

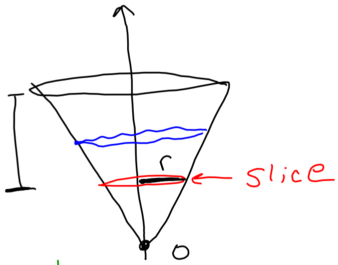
$\rho g = 62.5 \ \frac{lbs}{ft^3}$

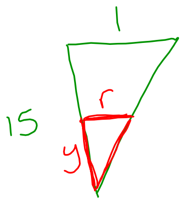
$F_{slice} = 8(62.5)\, y\, dy$

$$W = \int_0^3 8(62.5)\, y(3-y)\, dy$$

13. A tank in the shape of cone with radius 1 inch and height 15 ~~inches~~ feet is full of water to a depth of 7 ~~inches~~ feet. Set up but do not evaluate an integral done in pumping the water ~~through the spout~~. to the top. CLEARLY MARK YOUR AXIS AND WHAT DIRECTION IS POSITIVE!







$V_{slice} = \pi r^2 \, dy$

$\frac{1}{15} = \frac{r}{y}$

$V_{slice} = \pi \left(\frac{1}{15} y\right)^2 dy$

$r = \frac{1}{15} y$

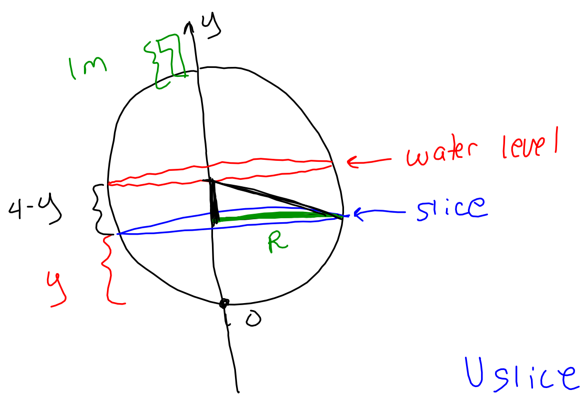
$F_{slice} = 62.5\pi \left(\frac{1}{15} y\right)^2 dy$

$\rho g = 62.5 \ \frac{lbs}{ft^3}$

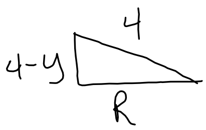
$d = 15 - y$

$W = \int_0^7 62.5\pi \left(\frac{1}{15} y\right)^2 (15 - y) \, dy$

12. A tank in the shape of sphere with radius 4 m is half full of water. The water is pumped from a spout at the top of the tank that is 1 m high. Set up but do not evaluate an integral done in pumping the water through the spout. CLEARLY MARK YOUR AXIS AND WHAT DIRECTION IS POSITIVE!



$V_{slice} = \pi R^2 \, dy$



$16 = (4-y)^2 + R^2$

$\cancel{16} = \cancel{16} - 8y + y^2 + R^2$

$R^2 = 8y - y^2$

$V_{slice} = \pi (8y - y^2) \, dy$

$d = 8 - y + 1 = 9 - y$

$W = \int_0^4 \pi (8y - y^2)(9800)(9 - y) \, dy$