Def • If $\Sigma |a_n|$ converges, then Σa_n is called absolutely convergent

• If $\Sigma |a_n|$ diverges, but Σa_n converges, then Σa_n is called conditionally convergent

Section 11.6

1. Determine whether the following series converge absolutely, convege conditionally, or diverge.

TD Fails! a.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ "absolute convergence test"

Testing $\Sigma \left| \frac{(-1)^n}{\sqrt{n}} \right|$ for convergence

$\Sigma \left| \frac{(-1)^n}{\sqrt{n}} \right| = \Sigma \frac{1}{\sqrt{n}}$ diverges by p-series $p = \frac{1}{2} < 1$

Thus $\Sigma \frac{(-1)^n}{\sqrt{n}}$ is not absolutely convergent, but it may still converge conditionally.

Do AST → show ① $c_n = \frac{1}{\sqrt{n}}$ is decreasing ✓②

and $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓③

$\boxed{\Sigma \frac{(-1)^n}{\sqrt{n}} \text{ converges conditionally}}$

b.) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ TD $\lim_{n\to\infty} \frac{(-1)^n n}{n+1}$ dne $\neq 0$ (by oscillation)

$\boxed{\text{diverges}}$

c.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+n}$ TD Fails!

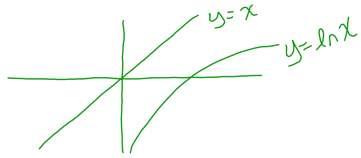
Look at $\Sigma \left| \frac{(-1)^n}{n^3+n} \right| = \Sigma \frac{1}{n^3+n} \leq \Sigma \frac{1}{n^3}$

converges by p-series $p=3>1$

larger converges so does smaller by CT

$\boxed{\Sigma \frac{(-1)^n}{n^3+n} \text{ converges absolutely!}}$

d.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ TD Fails!



Look at $\sum \left| \frac{(-1)^n}{n(\ln n)^2} \right| = \sum \frac{1}{n(\ln n)^2}$

IT $\int_2^{\infty} \frac{dx}{x(\ln x)^2}$ positive decreasing

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\ln x \le x$

$\int \frac{1}{u^2}\,du = -\frac{1}{u} = -\frac{1}{\ln x}$

$-\frac{1}{\ln x}\Big|_2^{\infty} = 0 + \frac{1}{\ln 2}$ finite!

$\sum \frac{(-1)^n}{n(\ln n)^2}$ converges absolutely.

e.) $\sum_{n=2}^{\infty} \frac{\cos n}{n^4}$

$\sum \left| \frac{\cos n}{n^4} \right| \le \sum \frac{1}{n^4}$

larger converges by p-series $p = 4 > 1$

$\sum \frac{\cos n}{n^4}$ converges absolutely!

f.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{6n+2}$

$\sum \left| \frac{(-1)^n}{6n+2} \right| = \sum \frac{1}{6n+2} \le \sum \frac{1}{6n}$

divergent p-series

CT fails

larger diverges, CT fails.

CT Fails, Try LCT

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ and finite

then $\sum a_n$ & $\sum b_n$ do the same thing.

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{6n+2}}{\frac{1}{6n}} = \lim_{n\to\infty} \frac{6n}{6n+2} = 1 > 0$

Since $\sum \frac{1}{6n}$ diverges, so does $\sum \frac{1}{6n+2}$.

$\sum \frac{(-1)^n}{6n+2}$ does not converge absolutely, but it still may converge conditionally

now do AST $c_n = \frac{1}{6n+2}$ decreases

$\lim_{n\to\infty} \frac{1}{6n+2} = 0$

$\sum \frac{(-1)^n}{6n+2}$ converges conditionally!

g.) $\sum_{n=1}^{\infty} \frac{(-9)^n}{5^n n^{10}}$

RT: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$ $>1 \to$ diverges; $<1 \to$ converges absolutely; $=1 \to$ Test fails

$$\lim_{n\to\infty} \left| \frac{(-9)^{n+1}}{5^{n+1}(n+1)^{10}} \cdot \frac{5^n n^{10}}{(-9)^n} \right| = \lim_{n\to\infty} \left| \frac{(-9)\cancel{(-9)^n}}{(5)\cancel{5^n}(n+1)^{10}} \cdot \frac{\cancel{5^n}\, n^{10}}{\cancel{(-9)^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-9n^{10}}{5(n+1)^{10}} \right| = \frac{9}{5} > 1$$

$\boxed{\text{diverges}}$

h.) $\sum_{n=1}^{\infty} \frac{(-7)^n}{(n+2)!}$ $\lim_{n\to\infty} \left| \frac{(-7)^{n+1}}{(n+3)!} \cdot \frac{(n+2)!}{(-7)^n} \right|$

$$\lim_{n\to\infty} \left| \frac{(-7)\cancel{(-7)^n}}{(n+3)\cancel{(n+2)!}} \cdot \frac{\cancel{(n+2)!}}{\cancel{(-7)^n}} \right| = 0 < 1$$

converges absolutely!

2. Explain why the Ratio Test is or is not conclusive (you decide which) for the following series:

a.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{n+3} \cdot \frac{n+2}{(-1)^n} \right| = 1 \to \text{RT Fails!}$$

b.) $\sum_{n=1}^{\infty} \frac{(-7)^n n!}{(2n+1)!}$ will be conclusive

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$$

$$\ln n < n^p < a^n < n! < n^n$$

Section 11.8

3. For the following power series, find the radius and interval of convergence.

• interval that contains all values of x for which the series converges.

• radius of convergence is $R = \frac{1}{2}(\text{length of interval})$

a.) $\sum_{n=1}^{\infty} \frac{(-4)^n x^n}{n^2+5}$

center = 0.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-4)^{n+1}x^{n+1}}{(n+1)^2+5}\cdot\frac{n^2+5}{(-4)^n x^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{-4\cancel{(-4)^n}\, x\, \cancel{x^n}}{(n+1)^2+5}\cdot\frac{n^2+5}{\cancel{(-4)^n}\cancel{x^n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{-4x(n^2+5)}{(n+1)^2+5}\right|$$

$$= |-4x|$$

$|x-c| < \underline{\underline{R}}$

$$= |-4||x|$$

$$= 4|x|$$

$\boxed{R = \frac{1}{4}}$

div R R div

$-\frac{1}{4}$ 0 $\frac{1}{4}$

center

$4|x| < 1 \longrightarrow |x| < \frac{1}{4}$

$-\frac{1}{4} < x < \frac{1}{4}$

test endpoints for convergence

$\sum \frac{(-4)^n x^n}{n^2+5}$ $x = \frac{1}{4}$: $\sum \frac{(-4)^n\left(\frac{1}{4}\right)^n}{n^2+5} = \sum \frac{(-1)^n}{n^2+5}$

include in interval

converges by AST since $C_n = \frac{1}{n^2+5}$ decreases to 0.

$-\frac{1}{4}$ 0 $\frac{1}{4}$

$x = -\frac{1}{4}$: $\sum \frac{(-4)^n\left(-\frac{1}{4}\right)^n}{n^2+5}$

also include

$= \sum \frac{1}{n^2+5} < \sum \frac{1}{n^2}$ converges by p-series $p = 2 > 1$

Answer: $R = \frac{1}{4}$

$I = \left[-\frac{1}{4}, \frac{1}{4}\right]$

b.) $\sum_{n=1}^{\infty} \frac{(-1)^n(3x-1)^n}{\sqrt{n}}$

center = $\frac{1}{3}$ $\left|x - \frac{1}{3}\right| < \underline{\underline{R}}$

RT $\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(3x-1)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{(-1)^n(3x-1)^n}\right|$

$$\lim_{n\to\infty}\left|\frac{\cancel{(-1)^n}(-1)\cancel{(3x-1)^n}(3x-1)}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{\cancel{(-1)^n}\cancel{(3x-1)^n}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(-1)(3x-1)\sqrt{n}}{\sqrt{n+1}}\right| = |(-1)(3x-1)|$$

$$= |3x-1|$$

$|3x-1| < 1$

$\boxed{R = \frac{1}{3}}$

$\left|3\left(x-\frac{1}{3}\right)\right| < 1 \quad \left|x - \frac{1}{3}\right| < \frac{1}{3}$

$-\frac{1}{3} < x - \frac{1}{3} < \frac{1}{3}$

$-\frac{1}{3} + \frac{1}{3} < x < \frac{1}{3} + \frac{1}{3}$

$0 < x < \frac{2}{3}$

b.) $\sum_{n=1}^{\infty} \frac{(-1)^n(3x-1)^n}{\sqrt{n}}$

$x = 0$: $\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$

do not include

diverges p-series $p = \frac{1}{2} < 1$

$x = \frac{2}{3}$: $\sum \frac{(-1)^n(1)^n}{\sqrt{n}}$

AST converges

do include

$C_n = \frac{1}{\sqrt{n}}$ decr to 0.

$R = \frac{1}{3}$

$I: \left(0, \frac{2}{3}\right]$

c.) $\sum_{n=0}^{\infty} \frac{(2n)!(x+2)^n}{100^n}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(2n+2)!\,(x+2)^{n+1}}{100^{n+1}}\cdot\frac{100^n}{(2n)!\,(x+2)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2n+2)(2n+1)\cancel{(2n)!}\,\cancel{(x+2)^n}(x+2)}{100\cdot\cancel{100^n}}\cdot\frac{\cancel{100^n}}{\cancel{(2n)!}\,\cancel{(x+2)^n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2n+2)(2n+1)(x+2)}{100}\right| = \infty$$

unless $x=-2$

in which case limit $=0<1$ will converge

$$\boxed{\begin{aligned} I &= \{-2\} \\ R &= 0 \end{aligned}}$$

d.) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{(2n+1)!}$

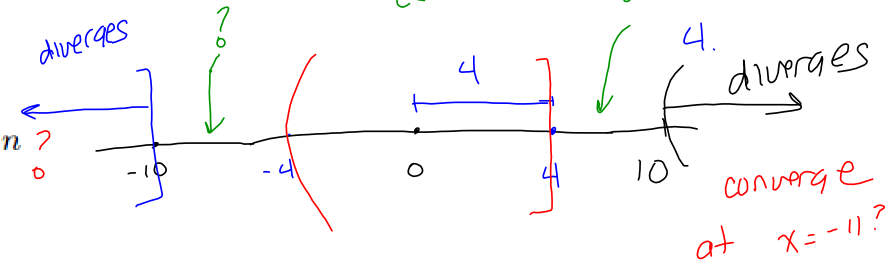
$I = (-\infty, \infty)$

$R = \infty$

4. Suppose it is known that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 4$ and diverges when $x = -10$. What can be said about the convergence or divergence of the following series, if anything?

center = 0

? R is at least 4.



converge at x = -11?

a.) $\sum_{n=0}^{\infty} c_n (8)^n$?

b.) $\sum_{n=0}^{\infty} c_n (-3)^n$ will converge

c.) $\sum_{n=0}^{\infty} c_n (12)^n$ diverges

d.) $\sum_{n=0}^{\infty} c_n (-4)^n$ do not know