Spring 2019 Math 152

Week in Review 9
courtesy: Amy Austin
(covering section 11.9, 11.10)

Section 11.9

1. Find a power series representation for the function and determine the radius of convergence.

   a.) $f(x) = \dfrac{1}{1-5x}$

   b.) $f(x) = \dfrac{3}{2+x}$

   c.) $f(x) = \dfrac{x^4}{8-x^3}$

   Theorem If $f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + ... + c_n x^n + ...$ has a radius of convergence R. Then

   a.) $f'(x) = c_1 + 2c_2 x + 3c_3 x^2 + ... + nc_n x^{n-1} + ...$ has a radius of convergence R.

   $$f'(x) = \sum_{n=1}^{\infty} c_n n x^{n-1} = \sum_{n=0}^{\infty} c_{n+1}(n+1)x^n$$

   b.) $\int f(x)\,dx = C + c_0 x + \dfrac{c_1 x^2}{2} + \dfrac{c_2 x^3}{3} ... + \dfrac{c_n x^{n+1}}{n+1} + ...$ has a radius of convergence R.

   $$\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{c_n}{n+1} x^{n+1}$$

   Note: When **differentiating** a power series, the starting index **will** change if the first term is constant. When **integrating** a power series, the starting index does **not** change under any circumstances.

2. If $f(x) = \sum_{n=0}^{\infty} \dfrac{n^4 (8x)^n}{n!}$, find $f'(x)$ and $\int f(x)\,dx$.

3. Find a power series representation for the function and determine the radius of convergence.

   a.) $\dfrac{1}{(5+2x)^2}$

   b.) Use part a.) to find a power series representation for the function $\dfrac{x^2}{(5+2x)^2}$

   c.) $\ln(2-x^2)$

   d.) Use part c.) to find a power series representation for the function $x\ln(2-x^2)$

   e.) Use part d.) to find a power series representation for the function $\int x\ln(2-x^2)\,dx$

   f.) $\arctan x$

   g.) Use part f.) to find a series representation for the function $\int_0^{1/2} \arctan\left(\dfrac{x}{3}\right) dx$

Section 11.10

The **Taylor Series** for $f(x)$ about $x = a$ is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

4. Find the Taylor Series for f centered at $a = -1$ if $f^{(n)}(-1) = \dfrac{(5)^n n!}{2^n (n+1)}$.

5. Find $f^{(50)}(2)$ if $f(x) = \sum_{n=0}^{\infty} \dfrac{10^{n+1}(x-2)^n}{(n+5)!}$, that is the 50^{th} derivative of f at $x = 2$.

6. Find the Taylor Series centered at $a = 4$ if $f(x) = \dfrac{1}{x^2}$

7. Find the Maclaurin Series for $f(x) = e^{-x^2}$.

8. Find the Maclaurin Series for $f(x) = x^2 \sin\left(\dfrac{x^4}{4}\right)$.

9. Evaluate the indefinite integral as an infinite series $\int \cos(x^{10})\,dx$.

10. Find the sum of the series $\sum_{n=0}^{\infty} \dfrac{(-1)^n (\pi)^{2n}}{4^{2n}(2n)!}$.

11. Find the sum of the series $\sum_{n=0}^{\infty} \dfrac{(-1)^n 2^n x^{3n}}{n!}$.