Spring 2020 Math 251

Week in Review I

courtesy: Amy Austin

(covering sections 12.1-12.5)

1. Find the equation of the sphere with center $(1, 4, 3)$ that touches the xy plane.

2. Does the sphere $x^2 + y^2 + z^2 + 4x - 2y - 8z = 5$ intersect the xz plane? If so, what is the intersection?

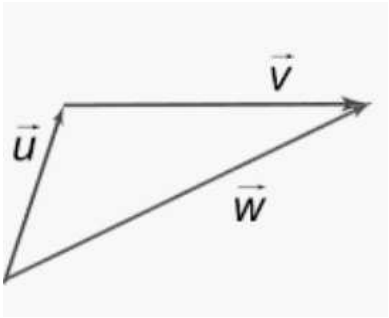
3. Find the equation of the sphere if one of their diameters has endpoints $(5, 1, 5)$ and $(7, 3, 9)$.

4. What does $y = 6 - x$ represent in $\mathbb{R}^3$?

5. What does $x^2 + z^2 = 16$ represent in $\mathbb{R}^3$?

6. Given $\mathbf{a} = \langle -7, 1, 2\rangle$ and $\mathbf{b} = \langle 5, -1, 1\rangle$, find a unit vector in the direction of $\mathbf{a} + 2\mathbf{b}$.

7. For the picture seen below, write $\mathbf{v}$ in terms of $\mathbf{u}$ and $\mathbf{w}$.



8. Compute $\mathbf{a} \cdot \mathbf{b}$ if

   a.) $\mathbf{a} = \langle 4, 5, -1\rangle$ and $\mathbf{b} = \langle 2, 1, 3\rangle$.

   b.) $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and $\theta = 120°$.

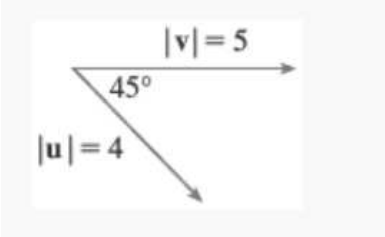
   c.) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$.

   d.) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and $\mathbf{a}$ is parallel to $\mathbf{b}$.

9. The points $A(0, -1, 6)$, $B(2, 1, -3)$ and $C(5, 4, 2)$ form a triangle. Find $\angle C$.

10. Find the cross product of $\langle 1, 1, 3\rangle$ and $\langle -2, -1, -5\rangle$.

11. Find $|\mathbf{u} \times \mathbf{v}|$ and determine if $\mathbf{u} \times \mathbf{v}$ points in or out of the page.



12. Find a vector that is orthogonal to the plane that passes through the points $P(1, 0, 1)$, $Q(2, 3, 4)$ and $R(2, 1, 1)$.

13. Find a vector equation of the line that passes through the point $(2, -5, 1)$ and is parallel to the vector $\langle 8, 10, -7\rangle$.

14. Find parametric equations and a symmetric equations for the line passing through the points $(-2, 3, 4)$ and $(5, 2, 8)$.

15. Do the lines $\dfrac{x-1}{2} = y = \dfrac{z-1}{4}$ and $x = \dfrac{y+2}{2} = \dfrac{z+2}{3}$ intersect?
    If so, what is the point of intersection?

16. Find an equation of the plane passing through the point $(3, 4, 5)$ and perpendicular to $\langle -1, 2, 5\rangle$.

17. Find the equation of the plane that passes through the points $P(1, 0, 1)$, $Q(2, 3, 4)$ and $R(2, 1, 1)$.

18. Find an equation of the plane passing through the point $(-3, 1, 4)$ and is perpendicular to the line $x = 2 - 3t$, $y = 3 - t$, $z = t$.

19. Find an equation of the plane passing through the point $(-1, -3, 2)$ that contains the line $x = -1 - 2t$, $y = 4t$, $z = 2 + t$.

20. Consider the planes $z = x + y$ and $2x - 5y - z = 1$.

    a.) Find the angle between the planes.

    b.) Find the line of intersecton of the planes.