Math 251 Engineering Math III
Spring 2020
Exam 1 Review
02/04/20

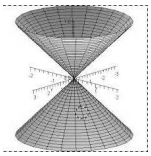
1. Consider the triangle formed by the points $P = (2,3,0)$, $Q = (4,-1,-1)$ and $R = (2,0,2)$.
   (a) Find the angle at vertex Q.
   (b) Find an equation of the plane containing P,Q, and R.
   (c) Find the area of the triangle formed by the points P,Q, and R.
2. Consider the plane P_1 given by the equation $2x - y + 3z = 7$ and the plane P_2 given by the equation $3x + y + 2z = 3$.
   (a) Are P_1 and P_2 parallel? Justify your answer.
   (b) Find a point (x_0, y_0, z_0) that lies on both planes.
   (c) Find a parametric equation for the line where the two planes intersect.
3. What is the equation for a sphere centered at $(3,4,12)$ of radius 6. Does it intersect the xz plane? If so, what is the intersection?
4. Let $\mathbf{a} = \langle 1,2,-2\rangle$ and $\mathbf{b} = \langle 2,-1,2\rangle$. Find $\text{Proj}_{\mathbf{b}}\mathbf{a}$.
5. Let $\mathbf{a} = \langle -2,2,1\rangle$. Find a vector $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ so that $\text{Comp}_{\mathbf{a}}\mathbf{b} = -4$.
6. Let $\mathbf{r}(t) = \langle t^2, \frac{t-1}{t^2-1}, \frac{\sin t}{t}\rangle$.
   (a) Find the domain of $\mathbf{r}(t)$.
   (b) Find $\lim_{t\to 1}\mathbf{r}(t)$.
7. Let $\mathbf{r}(t) = \langle \cos(t^2), \sin(t^2), t^2\rangle$.
   (a) Find $\mathbf{T}(\sqrt{\pi})$, the unit tangent vector at $t = \sqrt{\pi}$.
   (b) Find $\mathbf{a}(t)$, the acceleration vector, at time t.
   (c) Find the length of the curve from $(1,0,0)$ to $(1,0,2\pi)$.
   (d) Find the curvature of the curve traced out by $\mathbf{r}(t)$ when $t = \sqrt{\pi}$.

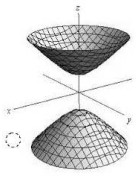
8. Match the equation with the images that follow.

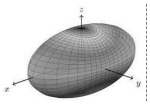
   Equation 1: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

   Equation 2: $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

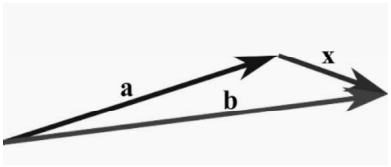
   Equation 3: $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.







9. Use the figure below to answer the questions that follow.



   (a) Write $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

   (b) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $\mathbf{a} \cdot \mathbf{b}$.

   (c) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $|\mathbf{a} \times \mathbf{b}|$ and determine whether $\mathbf{a} \times \mathbf{b}$ is directed into or out of the page.

10. Find parametric equations for the tangent line to the curve $x = 4\sqrt{t}$, $y = t^2 - 10$, $z = \dfrac{4}{t}$ at $(8, 6, 1)$.

11. If $\mathbf{r}'(t) = \left\langle t, e^t, te^{3t} \right\rangle$ and $\mathbf{r}(0) = \langle 1, 3, 2 \rangle$, find $\mathbf{r}(t)$.

12. Find $\displaystyle\int_0^1 \left(\frac{4t}{t^2 + 1}\mathbf{j} - \frac{1}{1 + t^2}\mathbf{k} \right) dt$.

13. Given the curves $\mathbf{r}_1(t) = \left\langle 3t, t^2, t^3 \right\rangle$ and $\mathbf{r}_2(v) = \langle \sin v, \sin(2v), 6v \rangle$ intersect at the origin, find the angle of intersection.

14. Find parametric equations for the line that passes through $(2, -1, 5)$ and is

    (a) parallel to the line $\dfrac{x+1}{3} = \dfrac{y-6}{4} = z$.

    (b) perpendicular to the plane $8x - 11y = 2z + 6$.

15. Consider the line that passes through the points $(4, 3, -1)$ and $(5, 3, 5)$. Where does this line intersect the three coordinate planes, and if it does not intersect all of the three coordinate planes, explain why not.

16. Find the equation of the plane that passes through the point $(1, 0, 1)$ and

    (a) is perpendicular to the line $x = 9 - t$, $y = 7 + 2t$, $z = t$.

    (b) contains line $x = 9 - t$, $y = 7 + 2t$, $z = t$.

17. Describe the space curve $x = \sin t$, $y = 3$, $z = \cos t$.