Math 251 Engineering Math III
Spring 2020
Exam 2 Review
03/02/20

1. Sketch the domain of $f(x, y) = \sqrt{x^2 - y}$ and describe the level curves.
2. Sketch the domain of $f(x, y) = \ln(y^2 + x^2 - 1)$ and describe the level curves.
3. What are the level surfaces to the equation $f(x, y, x) = x + y + z$?
4. For $f(x, y) = \sin(x^2 + y^2)$, find all first and second partial derivatives.
5. Find an equation for the tangent plane to the surface $z = 2x^2 + y^2$ at the point $(1, 1)$.
6. Find the tangent plane to the surface $2xy + 3yz + 7xz = -9$ at the point $(1, 2, -1)$.
7. If $z = x^3y^2$, find the differential, $dz$, and explain what it measures.
8. Consider a rectangular box with length $l$, width $w$ and height $h$. If $A$ is the surface area of the box, find the differential, $dA$.
9. The height of a cone was measured to be 3 cm with a maximum error of 0.1 cm and the radius of the cone is measured to be 2 cm with a maximum error of 0.2 cm. Use differentials to estimate the maximum error in the calculated volume of the cone.
10. Find the equation of the tangent plane to the surface $f(x, y) = xe^{xy}$ at the point $(1, 0, 1)$ and use this plane to approximate $f(1.1, -0.2)$.
11. Use a linear approximation to estimate $\left((2.1)^2 + (0.1)^3\right)^3$
12. Use differentials to approximate $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$.
13. If $z = e^{x^2+y^2}$, $x = e^t$. $y = \cos t$, find $\frac{dz}{dt}$.
14. For $z = xy$, $x = \cos(st^2)$, $y = \sin e^t$, find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$.
15. The radius and height of a circular cylinder change with time. When the height is 1 meter and the radius is 2 meters, the radius is increasing at a rate of 4 meters per second and the height is decreasing at a rate of 2 meters per second. At that same instant, find the rate at which the volume is changing.

16. Let $f(x, y) = \sqrt{xy}$. Find the directional derivative of f at the point $P(4, 1)$ in the direction from P to $Q(6, 2)$.
17. Let $f(x, y) = \sqrt{xy}$. What is the direction of the largest rate of change at the point $P(4, 1)$?
18. Let $f(x, y) = e^{x+y}$. What is the maximum rate of change at the point $P(-1, 1)$?
19. Find an equation for the tangent plane and normal line to the surface $x^2 - 5y^2 + z^2 = 4$ at the point $(7, 3, 0)$.
20. For the $f(x, y) = 2x^3 - xy^2 + 5x^2 + y^2 + 5$, find all local minima, maxima, and saddle points.
21. Find the absolute maximum and minimum values of $f(x, y) = 7 + xy - x - 2y$ over the closed triangular region with verticies $(1, 0)$, $(5, 0)$, $(1, 4)$.
22. Find the absolute maximum and minimum values of $f(x, y) = xy^2 + 2$ over the region $D = \{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 3\}$.
23. Use the method of Lagrange to find the maximum and minimum values of $f(x, y) = 6x + 6y$ subject to the constraint $x^2 + y^2 = 18$.
24. Use the method of Lagrange to find the maximum and minimum values of $f(x, y) = y^2 - x^2$ subject to the constraint $\frac{1}{4}x^2 + y^2 = 25$.
25. Use the method of Lagrange to find the extreme values of $f(x, y, z) = xyz$ subject to the constraint $2x + y + 3z = 60$.