Spring 2020 Math 251

Week in Review 3
courtesy: Amy Austin
(covering sections 14.5, 14.6)

1. If $y = x^4$ and $x = \tan t$, find $\dfrac{dy}{dt}$.

2. If $z = \ln(9x - 6y)$, $x = \cos(e^t)$, $y = \sin^3(4t)$, find $\dfrac{dz}{dt}$.

3. If $w = u^2 + 2uv$, $u = r \ln s$, $v = 2r + s$, find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$.

4. If $z = r^3 + s + v^2$, $r = xe^y$, $s = ye^x$, $v = x^2 y$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

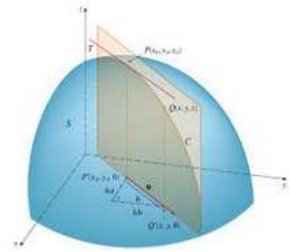
5. If $z = x^4 + xy^3$, $x = uv^3 + w^4$, $y = u + ve^w$, find find $\dfrac{\partial z}{\partial u}$ when $u = 1$, $v = 1$, $w = 0$.

6. If $u = x^4 y + y^2 z^3$, $x = 2rs + 3st$, $y = rs^2 t$, $z = r^2 e^{2t}$, find $\dfrac{\partial u}{\partial s}$ and $\dfrac{\partial u}{\partial t}$.

7. The height and radius of a right circular cone are changing with respect to time. If the base radius of the cone is increasing at a rate of $\dfrac{1}{4}$ inches per minute while the height is decreasing at a rate $\dfrac{1}{10}$ inches per minute, find the rate in which the volume if the cone is changing when the radius of the cone is 2 inches and the height of the cone is 1 inch.

8. The length l, width w and height h of a box change with time. At a certain instant, the dimensions are $l = 1$ m, $w = 3$ m and $h = 2$ m, and l and w are increasing at rate of 2 m/s while h is decreasing at a rate of 3 m/s. At that same instant, find the rate at which the surface area is changing.

Definition: The **directional derivative** of $z = f(x, y)$ at (x_0, y_0) in the direction of a unit vector $\mathbf{u} = \langle a, b\rangle$ is $D_{\mathbf{u}} f(x_0, y_0) = f_x(x_0, y_0)a + f_y(x_0, y_0)b = \langle f_x(x, y), f_y(x, y)\rangle \cdot \langle a, b\rangle$.



9. Given $f(x, y) = xy \sin x$, find the directional derivative at the point $\left(\dfrac{\pi}{2}, -1\right)$ in the direction $\mathbf{u} = \left\langle \dfrac{3}{5}, \dfrac{4}{5} \right\rangle$.

10. Given $f(x, y) = x^3 y^2$, find the directional derivative at the point $(-1, 2)$ in the direction $4\mathbf{i} - 3\mathbf{j}$.

11. If $f(x, y) = x^2 e^{xy}$, find the rate of change of f at the point $(1, 0)$ in the direction of the point $P(1, 0)$ to the point $Q\,(5, 2)$.

12. Find the gradient of $f(x, y) = x^2 + y^3 - 4xy$ at the point $(1, -1)$.

13. If $f(x, y) = x^2 e^{-2y}$, $P(2, 0)$, $Q(-3, 1)$.

    a.) Find the directional derivative at Q in the direction of P.

    b.) Find a vector in the direction in which f increases most rapidly at P, and find the rate of change of f in that direction.

14. Find the maximum rate of change of $f(x, y) = \sin^2(3x + 2y)$ at the point $\left(\dfrac{\pi}{6}, -\dfrac{\pi}{8}\right)$ and the direction in which it occurs.

15. Find the equation of the tangent plane to the surface $f(x, y) = x^2 + y^2 - 4xy$ at the point $(1, 2)$.

16. Find the equation of the tangent plane to the surface $xyz^3 = 8$ at the point $(2, 2, 1)$.

17. Find the equation of the tangent plane to the surface $xy + yz + zx = 5$ at the point $(1, 2, 1)$.