Spring 2020 Math 251

Week in Review 4
courtesy: Amy Austin
(covering sections 15.1)

Partial Integration and Iterated Integrals: Suppose $z = f(x,y)$ is a function of two variables that is integrable over the rectangle $R = [a,b] \times [c,d]$. This means $a \le x \le b$ and $c \le y \le d$.

(a) We use the notation $\displaystyle\int_a^b f(x,y)\,dx$ to mean that y is held fixed and $f(x,y)$ is integrated with respect to x from $x = a$ to $x = b$. This is called the **partial integration of** $f(x,y)$ **with respect to** x.

(b) We use the notation $\displaystyle\int_c^d f(x,y)\,dy$ to mean that x is held fixed and $f(x,y)$ is integrated with respect to y from $y = c$ to $y = d$. This is called the **partial integration of** $f(x,y)$ **with respect to** y.

(c) An **iterated integral** is an integral of the form $\displaystyle\int_c^d \int_a^b f(x,y)\,dxdy$ or $\displaystyle\int_a^b \int_c^d f(x,y)\,dydx$.

1. Find $\displaystyle\int_0^{\pi/4} x\sin(3y)\,dy$

2. Find $\displaystyle\int_1^e \frac{y\ln(x)}{x}\,dx$

3. Evaluate $\displaystyle\int_0^2 \int_0^3 (xy + x + y)\,dydx$ and $\displaystyle\int_0^3 \int_0^2 (xy + x + y)\,dxdy$

Fubini's Theorem: If f is continuous on the rectangle $R = [a,b] \times [c,d]$, then

(a) $\displaystyle\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dydx = \int_c^d \int_a^b f(x,y)\,dxdy.$

(b) In the case where $f(x,y) = g(x)h(y)$, then

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d g(x)h(y)\,dydx = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$$

4. $\displaystyle\int_{-3}^{3}\int_{0}^{\pi/2} (y + y^2 \cos x)\,dxdy$

5. Find $\displaystyle\iint_R \frac{x}{y^2}\,dA$, where $R = [0, 4] \times [1, 2]$

6. Find $\displaystyle\int_0^2 \int_0^1 (2x + 3y)^3\,dxdy$

7. Find $\displaystyle\iint_R e^{2x+y}\,dA$, where $R = [0, \ln 2] \times [0, \ln 3]$

8. Find $\displaystyle\iint_R (y \cos(xy))\,dA$, where $R = [0, 2] \times [0, \pi]$

9. Find $\displaystyle\iint_R x \sec^2 y\,dA$, where $R = \{(x, y)|0 \le x \le 2, 1 \le y \le \frac{\pi}{4}\}$

   FACT: If $f(x, y) \ge 0$ and f is continuous on the rectangle R, then the volume V of the solid that lies above R and under the surface $f(x, y)$ is $V = \displaystyle\iint_R f(x, y)\,dA$

10. Find the volume of the solid S that is bounded by the paraboloid $x^2 + y^2 + z = 16$, $z = 0$, $0 \le x \le 4$, $0 \le y \le 4$.