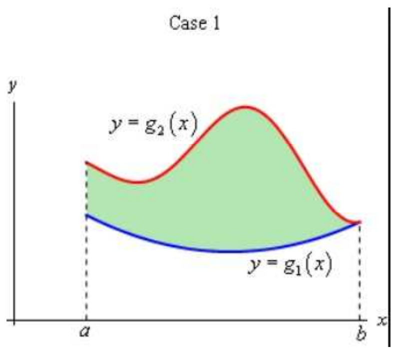
Spring 2020 Math 251

Week in Review 5

courtesy: Amy Austin

(covering sections 15.2-15.3)

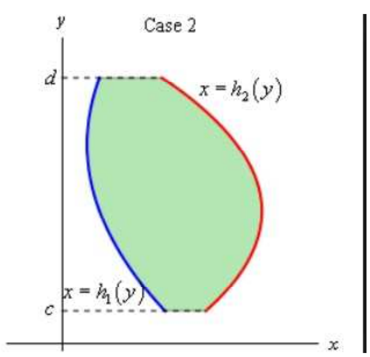
Type I: A plane region D is said to be of type I if it lies between the graphs of two continuous functions of x, that is $D = \{(x, y)|a \le x \le b, g_1(x) \le y \le g_2(x)\}$.



If f is continuous on a type I region $D = \{(x, y)|a \le x \le b, g_1(x) \le y \le g_2(x)\}$, then

$$\iint_R f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dydx$$

Type II: A plane region D is said to be of type II if it lies between the graphs of two continuous functions of y, that is $D = \{(x, y)|c \le y \le d, h_1(y) \le x \le h_2(y)\}$.



If f is continuous on a type II region $D = \{(x, y)|c \le y \le d, h_1(y) \le x \le h_2(y)\}$, then

$$\iint_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dxdy$$

1. Sketch the region of integration and evaluate $\iint_D xe^y \, dA$ where D is the region bounded by $y = 0$, $y = x^2$ and $x = 2$.

2. Sketch the region of integration and evaluate $\int_1^4 \int_1^{\sqrt{x}} (x + y) \, dydx$.

3. Sketch the region of integration and evaluate $\int_0^1 \int_0^y (3 + x^2 y) \, dxdy$

4. Set up but do not evaluate both a type I and type II integral for $\iint_D ye^x \, dA$, where D is the triangular region with vertices $(0, 0)$, $(1, 1)$ and $(2, 0)$.

5. Sketch the region of integration and change the order of integration.

   (i) $\int_0^4 \int_{\sqrt{y}}^2 f(x, y) \, dxdy$

   (ii) $\int_1^2 \int_0^{\ln x} f(x, y) \, dydx$

6. Set up but do not evaluate a double itegral that gives the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(1, 1)$, $(1, 2)$ and $(2, 1)$

7. Evaluate $\int_0^2 \int_x^2 e^{-y^2} \, dydx$

8. Evaluate $\int_0^2 \int_{y^2}^4 \sqrt{x} \sin x \, dydx$

Recall: If $P(x, y)$ is a point in the xy-plane, we can represent the point P in polar form: Let r be the distance from O to P and let θ be the angle between the polar axis and the line OP. Then the point P is represented by the ordered pair (r, θ), and r, θ are called the **polar coordinates** of P. Furthermore,

a.) $x = r\cos(\theta)$, $y = r\sin(\theta)$

b.) $\tan(\theta) = \frac{y}{x}$, thus $\theta = \arctan\left(\frac{y}{x}\right)$

c.) $x^2 + y^2 = r^2$

Change to Polar Coordinates in a Double Integral: If f is continuous on a polar rectangle R given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, where $0 \le \beta - \alpha \le 2\pi$, then

$$\iint_R f(x, y) \, dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta) r \, dr d\theta$$

9. Evaluate $\iint_R (x+2)\,dA$, where R is the region bounded by the circle $x^2+y^2=4$.

10. Evaluate $\iint_R 4y\,dA$, where R is the region in the second quadrant bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$.

11. Evaluate $\iint_R 3x^2\,dA$, where R is the region in the first quadrant enclosed by the by the circle $x^2+y^2=9$ and the lines $y=0$ and $y=x$.

12. Change $\int_0^3 \int_0^{\sqrt{9-x^2}} x^2\,dydx$ to a polar double integral. Do not evaluate

13. Change $\int_0^4 \int_0^{\sqrt{4x-x^2}} \sqrt{x^2+y^2}\,dydx$ to a polar double integral. Do not evaluate

14. Set up but do not evaluate a double integral that gives the volume of the solid that lies above the xy-plane, below the sphere $x^2+y^2+z^2=81$ and inside the cylinder $x^2+y^2=4$

15. Find the volume of the solid bounded by the paraboloids $z=20-x^2-y^2$ and $z=4x^2+4y^2$