EXAMPLE 6: Sketch the graph of $f(x) = \frac{x}{(x-1)^2}$ by locating intervals of increase/decrease, local extrema, concavity and inflection points.

$f(x) = \frac{x}{(x-1)^2}$  

Vertical Asymptote $x=1$

Horizontal Asymptote $y=0$

$f'(x) = \frac{-x-1}{(x-1)^3}$  

Critical number $0$ at $x=-1$

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f'</td>
<td></td>
<td>-</td>
<td></td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Local min at $(-\frac{1}{2}, -\frac{1}{4})$. No local max since $x=1$ is a V.A.

$f''(x) = \frac{2x+4}{(x-1)^4}$  

$f''(x) = 0$ if $x = -2$

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f''</td>
<td>+</td>
<td></td>
<td>+</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

Inflection point: $(-\frac{1}{2}, -\frac{2}{3})$
**Second derivative test for local extrema.** If \( x = c \) is a critical number for \( f(x) \), then:

- If \( f''(c) > 0 \), then \( f \) is concave up, therefore \( f(x) \) has a local minimum at \( x = c \).
- If \( f''(c) < 0 \), then \( f \) is concave down, therefore \( f(x) \) has a local maximum at \( x = c \).
- If \( f''(c) = 0 \) or does not exist, then the test fails, therefore use the first derivative test to find the local extrema.

**EXAMPLE 6:** Use the second derivative test to find the local extrema for \( f(x) = x^3 - 3x - 1 \).

**EXAMPLE 7:** Find a cubic function \( f(x) = ax^3 + bx^2 + cx + d \) that has a local maximum value of 3 at \(-2\) and a local minimum value of 0 at \(1\).

\[
\begin{align*}
f(x) &= ax^3 + bx^2 + cx + d \\
f'(x) &= 3ax^2 + 2bx + c \\
f''(x) &= 6ax + 2b \\
f(-2) = 3 &\Rightarrow -8a + 4b - 2c + d = 3 \\
f(1) = 0 &\Rightarrow a + b + c + d = 0 \\
f'(-2) = 0 &\Rightarrow 12a - 4b + c = 0 \\
f'(1) = 0 &\Rightarrow 3a + 2b + c = 0
\end{align*}
\]

On calculator: choose matrix \( \rightarrow \) edit \( \rightarrow \) matrix: \[
\begin{bmatrix}
-8 & 4 & -2 & 1 & 3 \\
1 & 1 & 1 & 1 & 0 \\
12 & -4 & 1 & 0 & 0 \\
3 & 2 & 1 & 0 & 0
\end{bmatrix}
\]

# rows # columns

Go back to home screen

Go to matrix \( \rightarrow \) math \( \rightarrow \) scroll to \( \rightarrow \) \rref

Call up matrix, enter.