

24. Use the method of Lagrange to find the maximum and minimum values of

$$f(x, y) = y^2 - x^2 \text{ subject to the constraint } \frac{1}{4}x^2 + y^2 = 25.$$

$$\langle -2x, 2y \rangle = \lambda \langle \frac{1}{2}x, 2y \rangle$$

$$-2x = \frac{1}{2}\lambda x \rightarrow -4x = \lambda x$$

$$0 = \lambda x + 4x$$

$$2y = 2\lambda y \leftarrow = x(\lambda + 4)$$

$$\frac{1}{4}x^2 + y^2 = 25 \quad \left\{ \begin{array}{l} x=0 \\ \lambda = -4 \end{array} \right.$$

$$\left\{ \begin{array}{l} y^2 = 25 \\ y = \pm 5 \end{array} \right.$$

$$(0, \pm 5)$$

$$\left\{ \begin{array}{l} \lambda y = -8y \\ y = 0 \end{array} \right.$$

$$x^2 = 100$$

$$\lambda = \pm 10$$

$$(\pm 10, 0)$$

$$f(0, \pm 5) = 25$$

$$f(\pm 10, 0) = -100$$

$$\text{Abs max} = 25$$

$$\text{Abs min} = -100$$

4 consideration points

at $(30, 0, 0)$, $(0, 60, 0)$, $(0, 0, 20)$, $f = 0 = \min$

at $(10, 20, \frac{20}{3}) = (10)(20)(\frac{20}{3}) = \frac{4000}{3} = \max$

25. Use the method of Lagrange to find the extreme values of

$f(x, y, z) = xyz$ subject to the constraint $2x + y + 3z = 60$.

$$\langle yz, xz, xy \rangle = \lambda \langle 2, 1, 3 \rangle$$

$$① yz = 2\lambda \leftarrow \text{if } \lambda = 0 \rightarrow \text{then } yz = 0$$

$$② xz = \lambda \leftarrow \text{if } \lambda = 0 \quad y=0 \quad z=0$$

$$③ xy = 3\lambda \leftarrow \text{if } \lambda = 0 \quad (30, 0, 0)$$

now, if $\lambda \neq 0$ multiply both sides of ① by x , ② by y , ③ by z then $x=0, z=0$
 $y=60$
 $(0, 60, 0)$

$$xyz = 2xz \quad \text{then } x=0, y=0$$

$$xyz = \lambda y \quad (0, 0, 20)$$

$$xyz = 3\lambda z$$

$$2x + 2x + 2x = 60$$

$$2x\lambda = xy \quad y = 2x$$

$$6x = 60$$

$$2x\lambda = 3xz \quad z = \frac{2x}{3}$$

$$x = 10$$

$$xy = 2x\lambda \quad x = \frac{1}{2}y$$

$$y + y + y = 60$$

$$xy = 3xz \quad z = \frac{1}{3}y$$

$$3y = 60 \quad y = 20$$

$$3xz = 2x\lambda, 3xz = \lambda y \quad x = \frac{3z}{2}, y = 3z$$

$$3z + 3z + 3z = 60 \quad 9z = 60 \quad z = \frac{60}{9} = \frac{20}{3}$$