Review for exam 1 will be Monday, Sept 26th 5:30-7:30 pm

Section 2.1

Compute the exact value of the following limits. If the limit does not exist, support your answer by examining the left and right hand limits.

1. \( \lim _{x \to 1} \frac{x^2 - 1}{x - 1} \)
   
   \( \frac{0}{0} \rightarrow \) Factor in common and simplify first
   
   \( \lim _{x \to 1} \left( x + 1 \right) = \lim _{x \to 1} (x) = 2 \)

2. \( \lim _{x \to 0} \frac{\sin(3x)}{x} \)
   
   \( \lim _{x \to 0} \frac{\sin(3x)}{x} = \lim _{x \to 0} \frac{3\sin(3x)}{3x} = 3 \cdot \lim _{x \to 0} \frac{\sin(3x)}{3x} = 3 \cdot 1 = 3 \)

3. \( \lim _{x \to 0} \frac{e^x - 1}{x} \)
   
   \( \lim _{x \to 0} \frac{e^x - 1}{x} = \lim _{x \to 0} \frac{e^x}{1} = e^0 = 1 \)

4. \( \lim _{x \to 0} \frac{\ln(x)}{x^2} \)
   
   \( \lim _{x \to 0} \frac{\ln(x)}{x^2} = \lim _{x \to 0} \frac{\frac{1}{x}}{2x} = \lim _{x \to 0} \frac{1}{2x^2} = \infty \)

5. \( \lim _{x \to 0} \frac{x^4 - x^2}{x^2} \)
   
   \( \lim _{x \to 0} \frac{x^4 - x^2}{x^2} = \lim _{x \to 0} \frac{x^2(x^2 - 1)}{x^2} = \lim _{x \to 0} (x^2 - 1) = 0 \)

6. \( \lim _{x \to \infty} \frac{\sqrt{x^2 + 1}}{x} \)
   
   \( \lim _{x \to \infty} \frac{\sqrt{x^2 + 1}}{x} = \lim _{x \to \infty} \frac{\sqrt{1 + \frac{1}{x^2}}}{1} = \frac{1}{1} = 1 \)

7. \( \lim _{x \to 0} \frac{\sin(3x)}{x} \)
   
   \( \lim _{x \to 0} \frac{\sin(3x)}{x} = \lim _{x \to 0} \frac{3\sin(3x)}{3x} = 3 \cdot \lim _{x \to 0} \frac{\sin(3x)}{3x} = 3 \cdot 1 = 3 \)

8. \( \lim _{x \to 0} \frac{e^x - 1}{x} \)
   
   \( \lim _{x \to 0} \frac{e^x - 1}{x} = \lim _{x \to 0} \frac{e^x}{1} = e^0 = 1 \)

9. \( \lim _{x \to 0} \frac{\ln(x)}{x^2} \)
   
   \( \lim _{x \to 0} \frac{\ln(x)}{x^2} = \lim _{x \to 0} \frac{\frac{1}{x}}{2x} = \lim _{x \to 0} \frac{1}{2x^2} = \infty \)

10. \( \lim _{x \to 0} \frac{\sqrt{x^2 + 1}}{x} \)
    
    \( \lim _{x \to 0} \frac{\sqrt{x^2 + 1}}{x} = \lim _{x \to 0} \frac{\sqrt{1 + \frac{1}{x^2}}}{1} = \frac{1}{1} = 1 \)

11. Sketch the graph of \( f(x) \) and determine where the function
    
    \[ f(x) = \begin{cases} 
    x & \text{if } x < 1 \\
    \frac{1}{x} & \text{if } x > 1
    \end{cases} \]

    \text{It is not continuous.}

    If \( x = 1 \),
    
    \( f(1-0) = 1 \)
    
    \( f(1+0) = 1 \)
    
    \( f(1) \) does not exist.
    
    \( f(x) \) is not continuous at \( x = 1 \) because
    
    \( \lim _{x \to 1^-} f(x) = 1 \) does not exist and
    
    \( \lim _{x \to 1^+} f(x) = 1 \) does not exist.
    
    Therefore, \( f(x) \) is not continuous at \( x = 1 \).
12. Which of the following functions has removable discontinuity at x = 0? If the discontinuity is removable, determine the limit. If not, explain why.

(a) \( f(x) = \frac{x^2 - 1}{x - 1} \)

Limit exists as \( x \to 0 \) and is continuous at \( x = 0 \). The limit is 1.

(b) \( f(x) = \frac{x^2 - 1}{x^2 - 4} \)

Limit exists as \( x \to 0 \) and is continuous at \( x = 0 \). The limit is 1/2.

(c) \( f(x) = \frac{1}{x^2} \)

Limit does not exist as \( x \to 0 \) and is not continuous at \( x = 0 \).

16. Find the values of c and d that make each function continuous on all real numbers. Then, find the limit of the function.

First, find the limit of each piece of the function.

- For \( x < 0 \), \( f(x) = x + c \)
  - \( \lim_{x \to 0^-} (x + c) = c \)

- For \( x > 0 \), \( f(x) = \frac{d}{x^2} \)
  - \( \lim_{x \to 0^+} \frac{d}{x^2} = \lim_{x \to 0^+} \frac{d}{x^2} = \infty \)

To make the function continuous, \( c = 0 \) and \( d = 0 \).

17. Compare the following limits.

(a) \( \lim_{x \to 0} \frac{\sin(3x)}{x} \)

L'Hopital's Rule:

- \( \lim_{x \to 0} \frac{\cos(3x) \cdot 3}{1} = 3 \)

(b) \( \lim_{x \to 0} \frac{\tan(x)}{x} \)

L'Hopital's Rule:

- \( \lim_{x \to 0} \frac{\sec^2(x) \cdot 1}{1} = 1 \)

(c) \( \lim_{x \to 0} \frac{\ln(1 + x)}{x} \)

L'Hopital's Rule:

- \( \lim_{x \to 0} \frac{\frac{1}{1 + x}}{1} = 1 \)

(d) \( \lim_{x \to 0} \frac{e^x - 1}{x} \)

L'Hopital's Rule:

- \( \lim_{x \to 0} \frac{e^x}{1} = 1 \)

(e) \( \lim_{x \to 0} \frac{\sin(x)}{x} \)

L'Hopital's Rule:

- \( \lim_{x \to 0} \frac{\cos(x) \cdot 1}{1} = 1 \)
10. Find all horizontal and vertical asymptotes of
\[ f(x) = \frac{x^2}{x(x+1)(x-1)} \]

\[ \text{VA: } x = \pm 1 \]

\[ \text{HA: } \lim_{x \to \pm\infty} \frac{x^2}{x^3} = \frac{1}{x} = 0 \]

\[ \text{V.A. } y = 1 \]