1. Find the linear approximation, \( L(x) \), for \( f(x) = \sqrt{x} \) at \( x = 9 \). Using this linear approximation, which of the following is an estimate for \( \sqrt{11} \)?

\[ a) \frac{8}{3} \quad b) \frac{11}{3} \quad c) \frac{23}{6} \quad d) \frac{21}{6} \quad e) \frac{10}{3} \]

Linear approximation of \( f(x) \) at \( x = a \):

\[
L(x) = f(a) + f'(a)(x-a)
\]

\[
= f(9) + f'(9)(x-9)
\]

\[
L(x) = 3 + \frac{1}{6}(x-9) \approx \sqrt{x} \quad \text{for} \ x \ \text{near} \ 9
\]

\[
3 + \frac{1}{6}(11-9) \approx \sqrt{11}
\]

\[
3 + \frac{1}{3} = \frac{10}{3}
\]

2. If \( f(x) = 3x \cos^2(x^2) \), find \( f'(0) \).

\[ a) \ 0 \quad b) -3 \quad c) 3 \quad d) 1 \quad e) -9 \]

\[
f(x) = 3x \cos^2(x^2)
\]

\[
f'(x) = \frac{d}{dx} \left( 3x \cos^2(x^2) \right)
\]

\[
= 3 \cos^2(x^2) + 3x \left( 2 \cos(x^2) \left( -\sin(x^2) \right) \right)
\]

\[
f'(0) = 3
\]
3. A particle is moving according to the equation of motion \( f(t) = t^4 - 4t + 1 \), where \( t \geq 0 \), where \( t \) is measured in seconds and \( f(t) \) is measured in feet. What is the acceleration of the particle at the instant when the particle is at rest?

a) \( \frac{0{ft}}{s^2} \)  

b) \( \frac{0{ft}}{s} \)  

c) \( 12\frac{ft}{s^2} \)  

d) \( 12\frac{ft}{s} \)  

e) \( -12\frac{ft}{s^2} \)

\[ v(t) = f'(x) = 4t^3 - 4 \]

\[ v(t) = 0 \quad \text{when} \quad t = 1 \]

\[ a(t) = f''(t) = v'(t) = 12t^2 \]

\[ a(1) = 12 \quad \text{ft/sec/sec} = \text{ft/sec}^2 \]

4. Two sides of a triangle are fixed at 4cm and 6cm and the angle between them is increasing at a rate of .02 radians per second. How fast is the area of the triangle increasing when the angle between them is \( \frac{\pi}{6} \)?

\[ a = \frac{.12}{\sqrt{3}} \]

b) \( \frac{.02}{6} \)  

c) \( \frac{.02}{6\sqrt{3}} \)  

d) .12  

e) 12\sin(.02)

\[ \text{given:} \quad \frac{d\theta}{dt} = 0.02 \quad \frac{\text{radians}}{\text{sec}} \]

\[ \text{Find} \quad \frac{dA}{dt} \]

\[ A = \frac{1}{2}bh = \frac{1}{2}(6)(4\sin\theta) \]

\[ A = 12\sin\theta \]

\[ \frac{dA}{dt} = 12\cos\theta \frac{d\theta}{dt} \]

\[ \frac{dA}{dt} = 12\left( \frac{\sqrt{3}}{2} \right)(0.02) \text{ cm}^2/\text{sec} \]
5. Let \( f(x) = (1 + x^2)^{\frac{3}{2}} \). Then \( f''(0) = \)

a) 3  
b) 0  
c) 6  
d) \( \frac{3}{4\sqrt{2}} \)  
e) \( \frac{3}{4} \)

\[
\begin{align*}
  f'(x) &= \frac{3}{2} (1 + x^2)^{\frac{1}{2}} \cdot (2x) \\
  f''(x) &= \frac{3}{2} \frac{3(1 + x^2)}{(x)} \cdot \frac{1}{x} \\
  f''(x) &= g'h + gh' \\
  f''(x) &= \frac{3}{2} \frac{(1 + x^2)}{(x)} \cdot (2x) \cdot (x) + \frac{3(1 + x^2)}{(x)} \cdot (1) \\
  f''(0) &= 3
\end{align*}
\]

6. The function \( f(x) = x^3 + 5x - 1 \) is one-to-one. Let \( g = f^{-1} \). Then \( g'(5) = \)

a) 8  
b) \( \frac{1}{80} \)  
c) \( \frac{8}{25} \)  
d) \( \frac{1}{8} \)  
e) 80

\[
\begin{align*}
  f(x) &= x^3 + 5x - 1 \\
  f'(x) &= 3x^2 + 5 \\
  f'(1) &= 8
\end{align*}
\]

\[
\begin{align*}
  g'(a) &= \frac{1}{f'(g(a))} \\
  g'(5) &= \frac{1}{f'(g(5))} \\
  \quad a = 5 \\
  \text{What is } g(5) \text{?} \\
  \text{solve } f(x) &= 5 \\
  x^3 + 5x - 1 &= 5 \\
  x &= 1 \implies g(5) = 1
\end{align*}
\]
7. Given the curve parametrized by $x = t^3 - 3t^2 - 9t + 1$, $y = t^3 + 3t^2 - 9t + 1$, at which point does the curve have a vertical tangent?

a) $(1, -3)$  
b) $(6, 12)$  
c) $(-10, 6)$  
d) $(-1, 3)$  
e) $(1, 1)$

The slope of a parametric curve at $t = a$ is

$$m = \frac{dy/dt}{dx/dt} \bigg|_{t=a}$$

$$x = t^3 - 3t^2 - 9t + 1$$
$$y = t^3 + 3t^2 - 9t + 1$$

$$t = -1 \quad \begin{cases} x = -1 - 3 + 9 + 1 = 6 \\ y = -1 + 3 + 9 + 1 = 12 \end{cases}$$

$$t = 3 \quad \begin{cases} x = 27 - 27 - 27 + 1 = -26 \\ y = 27 + 27 - 27 + 1 = 28 \end{cases}$$

Solve $\frac{dx}{dt} = 0$:

$$3t^2 - 6t - 9 = 0$$
$$3(t^2 - 2t - 3) = 0$$
$$3(t - 3)(t + 1) = 0$$

Solution: $(6, 12) \pm (-26, 28)$

Only b is correct answer.

8. \[ \lim_{x \to 0} \frac{4 \cos x - 4 + 3 \sin x}{3x} = \]

a) $\frac{4}{3}$  
b) $-\frac{4}{3}$  
c) $\frac{3}{5}$  
d) 1  
e) 0

\[ \lim_{x \to 0} \frac{\sin \theta}{\theta} = 1 \quad \text{and} \quad \lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0 \]

\[ \lim_{x \to 0} \left( \frac{4 \cos x - 4}{5x} + \frac{3 \sin x}{5x} \right) = \frac{1}{5} \lim_{x \to 0} \left( \frac{4(\cos x - 1)}{x} + \frac{3 \sin x}{x} \right) \]

\[ = \frac{1}{5} (0 + 3) \]

\[ = \frac{3}{5} \]
9. Find the slope of the line tangent to the curve given by \( y^2 + xy = 8 \) at the point \((-2, -2)\).

   a) \(-2\)  
   b) \(-\frac{10}{3}\)  
   c) \(-\frac{1}{3}\)  
   d) \(-3\)  
   e) 0

\[ y^2 + xy = 8 \]  
Implicit diff:

\[ ay \frac{dy}{dx} + (1)y + x \frac{dy}{dx} = 0 \]

\[ \frac{dy}{dx} = \frac{-y}{2y + x} \]

\[ m = \frac{-(-2)}{2(-2) - 2} = \frac{2}{-4 - 2} = \frac{-1}{3} \]

10. Which of the following statements is true about the curve \((2 + \cos t)i + (1 + \sin t)j\)?

   a) Clockwise movement around the circle \((x - 2)^2 + (y - 1)^2 = 1\)

   b) Counterclockwise movement around the circle \((x - 2)^2 + (y - 1)^2 = 1\)

   c) Clockwise movement around the ellipse \(x^2/4 + y^2 = 1\)

   d) Counterclockwise movement around the ellipse \(x^2/4 + y^2 = 1\)

   e) None of the above statements is correct.

\[ \begin{align*}
  x &= 2 + \cos t \quad \Rightarrow \quad \cos t = x - 2 \\
  y &= 1 + \sin t \quad \Rightarrow \quad \sin t = y - 1 \\
  \cos^2 t + \sin^2 t &= 1 \\
  (x - 2)^2 + (y - 1)^2 &= 1 \\
  t &= 0 \quad \Rightarrow \quad x = 3 \\
  y &= 1
\end{align*} \]
11. Let \( f(x) \) be a differentiable function and let \( g(x) = 3x^2 - 1 \). Let \( H(x) = f(g(x)) \), the composite of \( f \) and \( g \). If \( f(0) = 1, f'(0) = -1, f(1) = 3, f'(1) = 2, f(2) = -1, f'(2) = 5 \), find \( H'(1) \).

\[ H(x) = f(g(x)) \]

\[ H(x) = f(3x^2 - 1) \]

Find \( H'(1) \)

by chain rule,

\[ H'(x) = f'(3x^2 - 1)(6x) \]

\[ H'(1) = f'(2)(6) \]

\[ H'(1) = 5(6) = 30 \]

\[ 12. \lim_{x \to \infty} 3^{1-x} = \]

a) 0 \hspace{2cm} b) \infty \hspace{2cm} c) -\infty \hspace{2cm} d) 1 \hspace{2cm} e) 3

\[ \lim_{x \to \infty} 3^{1-x} = \]

\[ \lim_{x \to \infty} (1-x) \]

\[ \lim_{x \to \infty} 3 = \frac{1}{3^\infty} = 0 \]
13. Find the inverse of \( f(x) = \frac{3x - 5}{7x + 2} \)

a) \( f^{-1}(x) = \frac{7x + 2}{3x - 5} \)

b) \( f^{-1}(x) = \frac{2x - 5}{3x + 7} \)

c) \( f^{-1}(x) = \frac{2x + 5}{7x + 3} \)

d) \( f^{-1}(x) = \frac{7x + 2}{3x - 5} \)

e) None of the above is correct.

\[
\begin{align*}
\text{Let } y &= \frac{3x - 5}{7x + 2} \\
x &= \frac{3y - 5}{7y + 2} \\
x(7y + 2) &= 3y - 5 \\
7xy + 2x &= 3y - 5 \\
2x + 5 &= 3y - 7xy \\
2x + 5 &= y(3 - 7x) \\
y &= \frac{2x + 5}{3 - 7x}
\end{align*}
\]

14. If \( (\cos 3t, t) \) is the position of an object at time \( t \), find the acceleration of the object at time \( t = \frac{\pi}{9} \).

a) \( \left\langle \frac{1}{2}, 0 \right\rangle \)

b) \( \left\langle -\frac{1}{2}, 0 \right\rangle \)

c) \( \left\langle -\frac{9}{2}, 0 \right\rangle \)

d) \( \left\langle \frac{9}{2}, 0 \right\rangle \)

e) \( (3, 0) \)

\[
\begin{align*}
\mathbf{r}(t) &= \langle \cos(3t), t \rangle \\
\mathbf{v}(t) &= \langle -3 \sin(3t), 1 \rangle \\
\mathbf{a}(t) &= \langle -9 \cos(3t), 0 \rangle \\
\mathbf{a}\left(\frac{\pi}{9}\right) &= \langle -9 \cos\left(\frac{\pi}{3}\right), 0 \rangle \\
&= \langle -9 \cdot \frac{1}{2}, 0 \rangle
\end{align*}
\]
15. If $f(x) = e^{x \tan x}$, find $f'(x)$.

a) $f'(x) = e^{x \tan x}$

b) $f'(x) = \sec^2 x e^{x \tan x}$

c) $f'(x) = (\tan x + x \sec^2 x) e^{x \tan x}$

d) $f'(x) = (\tan x + x \sec x \tan x) e^{x \tan x}$

e) $f'(x) = x \tan x e^{x \tan x - 1}$

16. Find the equation of the tangent line to the graph of $x = e^{2t}$, $y = te^t$ at the point $(1,0)$.

a) $y = 2x - 1$

b) $y = 4x - 4$

c) $y = \frac{1}{2}x - \frac{1}{2}$

d) $y = \frac{1}{3}x - \frac{1}{3}$

e) $y = x - 1$
17. Find the quadratic approximation for \( f(x) = \frac{1}{x} \) at \( x = 1 \).

   a) \( x^2 - 3x + 3 \)
   b) \( x^2 - x + 2 \)
   c) \( x^2 - 2x + 1 \)
   d) \( x^2 + 4x + 5 \)
   e) \( x^2 + x - 3 \)

\[
\Theta(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2
\]

\[
\Theta(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2
\]

\[
f = \frac{1}{x}
\]

\[
f' = \frac{-1}{x^2}
\]

\[
f'' = \frac{2}{x^3}
\]

\[
\Theta(x) = 1 - (x-1) + (x-1)^3
\]

\[= 1 - x + 1 + x^2 - 2x + 1
\]

\[= x^2 - 3x + 3
\]

18. The position of a particle is given by \( \mathbf{r}(t) = \left( \frac{\cos t}{e^t}, \frac{\sin t}{e^t} \right) \). Find the velocity and speed of the particle when \( t = 0 \).

\[
\mathbf{r}(t) = \left( \frac{\cos t}{e^t}, \frac{\sin t}{e^t} \right) = \left( e^{-t}\cos t, e^{-t}\sin t \right)
\]

\[
\mathbf{v}(t) = \left( -e^{-t}\cos t + e^{-t}(-\sin t), -e^{-t}\sin t + e^{-t}\cos t \right)
\]

\[
\mathbf{v}(0) = \left( -1, 1 \right) \text{ velocity, speed} = \left| \left( -1, 1 \right) \right|
\]

\[= \sqrt{(-1)^2 + (1)^2}
\]

\[= \sqrt{2}
\]
19. The radius of a sphere was given to be 8 inches with a maximum possible error in measurement of 0.01 inches. Find the differential \( dV \), and use it to estimate the maximum error in the calculated volume of the sphere.

\[
\text{def: } y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)
\]

\[
\Delta V = V(8.01) - V(8) \\
dV \approx \Delta V
\]

\[
v = \frac{4}{3} \pi r^3
\]

\[
\frac{dv}{dr} = \frac{4}{3} \pi (3r^2)
\]

\[
\frac{dv}{dr} = 4\pi r^2 \Rightarrow dv = 4\pi r^2 dr
\]

\[
= 4\pi (64)(0.01) \text{ in}^3
\]
20. Find all values of $x$ between $0$ and $2\pi$ where the tangent line to $f(x) = 2x - \tan x$ is horizontal.

\[
\text{solve } \quad f'(x) = 0 \\
2 - \sec^2 x = 0 \\
\sec^2 x = 2 \\
\sec x = \pm \sqrt{2} \Rightarrow \cos x = \pm \frac{1}{\sqrt{2}} \\
\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}} \quad \cos x = \pm \frac{\sqrt{2}}{2}
\]
21. A trough is 20 feet long. The end of the trough is an isosceles triangle with height 10 feet and length of 3 feet across the top. If water is poured in the trough at a rate of 3 cubic feet per minute, how fast is the water level rising when the height of the water is 1 foot?

\[
\frac{b}{h} = \frac{3}{10}
\]

Find \( \frac{dh}{dt} \) when \( h = 1 \) foot.

\[
V = \left( \text{Area triangle} \right) \times \text{(length)}
\]

\[
V = \left( \frac{1}{2} bh \right) (20) = 10bh
\]

\[
V = 10bh
\]

\[
v = 10 \left( \frac{3}{10} h \right) h
\]

\[
v = 3h^2
\]

\[
\frac{dv}{dt} = 6h \frac{dh}{dt}
\]

\[
3 = 6(1) \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{1}{2} \frac{f}{m}
\]
22. Find the derivative of the following functions:

a.) \( f(x) = \cos^3(\tan(x)) \)

b.) \( h(x) = \frac{x^3}{(5x + 8)^2} \)

\[
f'(x) = 3(\cos(\tan(x)))^2 \cdot (-\sin(\tan(x)) \sec^2 x)
\]

\[
h'(x) = \frac{3x^2(5x + 8) - 7x^3}{9(5x + 8)^2} + \frac{x^3[-7(5x + 8)]}{9(5x + 8)^2}
\]
23. Given the equation \(2e^{xy} = x + y\), find \(\frac{dy}{dx}\) when \(x = 0\) and \(y = 2\).

\[
\text{Find } \frac{dy}{dx} : \quad 2e^{xy} = 1 + \frac{dy}{dx}
\]

\[
2 \left( y + x \frac{dy}{dx} \right) e^{xy} = 1 + \frac{dy}{dx}
\]

\[
2 \left( \frac{dy}{dx} + 0 \right) e^{xy} = 1 + \frac{dy}{dx}
\]

\[
4 = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 3
\]
24. Find the points where the tangent line to the curve \( y = x^2 + x \) pass through the point (2, -3).

\[ f(x) = x^2 + x = x(x + 1) \]

\[ f'(x) = 0 \]
\[ 2x + 1 = 0 \]
\[ x = -\frac{1}{2} \]
\[ y = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \]

Wrong: \( m = \frac{dy}{dx} \bigg|_{x=2} \) not right because \((2, -3)\) is not on the parabola.

Since line is tangent to
\[ f(x) = x^2 + x + a^2 \]
\[ x = a, m = f'(a) \]
\[ f'(x) = 2x + 1 \]
\[ f'(a) = 2a + 1 \]

solve
\[ \frac{a^2 + a + 3}{a - 2} = 2a + 1 \]

\[ a^2 + a + 3 = (2a + 1)(a - 2) \]
\[ a^2 + a + 3 = 2a^2 - 3a - 2 \]
\[ 0 = a^2 - 4a - 5 \]
\[ = (a - 5)(a + 1) \]

Points: \[ f(x) = x^2 + x \]
\[ (5, 30), (-1, 0) \]
25. A rope is attached to the bow of a boat coming in for the evening. Assume the rope is drawn in over a pulley 5 feet higher than the bow at a rate of 2 feet per second. How fast is the boat docking when the length of the rope from the bow to the pulley is 13 feet?

\[ r^2 = b^2 + 25 \]
\[ a \frac{dr}{dt} = b \frac{db}{dt} \]

Given: \( \frac{dr}{dt} = -2 \frac{f}{s} \)

Find \( \frac{db}{dt} \) \( \left| r = 13 \right| \Rightarrow 13 = b^2 + 25 \)

\[ 169 - 25 = b^2 \]
\[ 144 = b^2 \]
\[ 12 = b \]

\[ (13)(-2) = 12 \frac{db}{dt} \Rightarrow \frac{db}{dt} = -\frac{26}{12} \frac{f}{s} \]

Docking at a rate of \( \frac{26}{12} \frac{f}{s} \)
Find the \(102^{nd}\) derivative of \(f(x) = \frac{1}{x^2}\)

\[
\begin{align*}
f(x) &= \frac{1}{x^2} \\
f'(x) &= -2x^{-3} \\
f''(x) &= -6x^{-4} \\
f'''(x) &= 24x^{-5} \\
f^{(4)}(x) &= -120x^{-6} \\
\vdots \\
f^{(102)}(x) &= (-1)^{102}(103)!x^{-104} \\
f^{(103)}(x) &= \frac{(-1)^{103}(104)!x^{-105}}{104!} \\
\end{align*}
\]