Section 2.5: Continuity

<u>Definition</u> We say $f(x)$ is continuous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$. Note that in order for this definition to be met, the following conditions must hold:

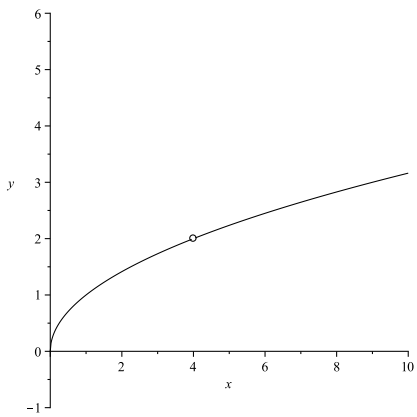
(a) $x = a$ is in the domain of $f(x)$ (This ensures that $f(a)$ is defined).

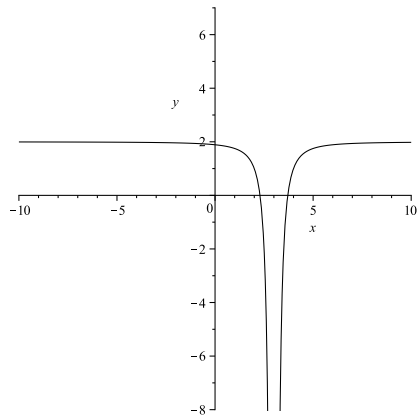
(b) $\lim_{x\to a} f(x)$ must exist. Thus $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$.

(c) $\lim_{x\to a} f(x) = f(a)$.

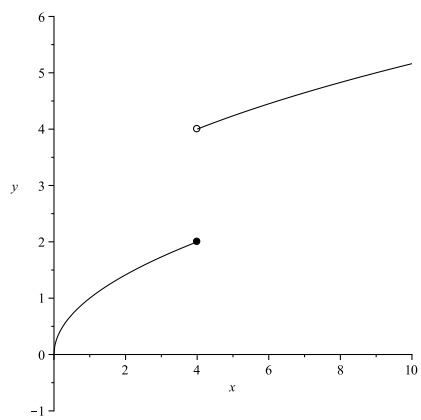
Some types of discontiniuties include 'holes', 'jumps' and 'vertical asumptotes'.

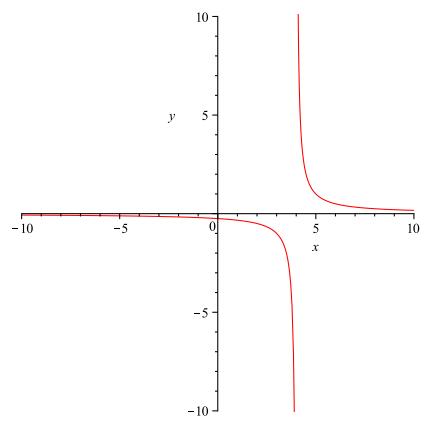
The graphs below represents a discontinuity at $x = 4$ because $f(4)$ is not defined.



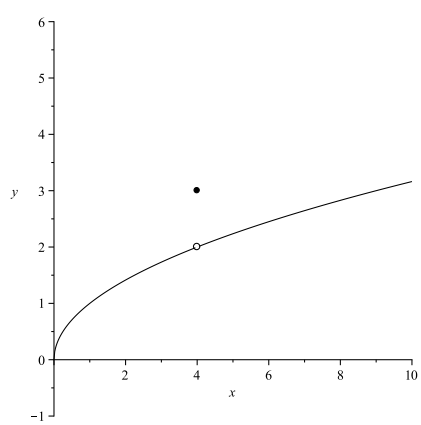


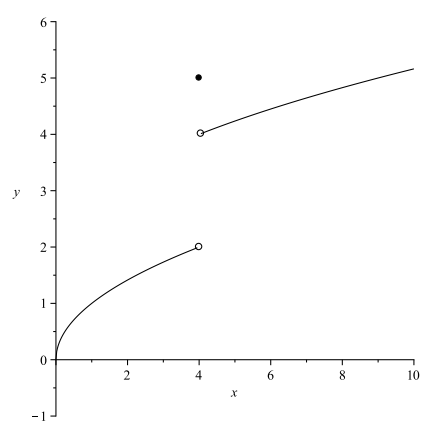
The graphs below represents a discontinuity at $x = 4$ because $\lim_{x\to 4} f(x)$ does not exist



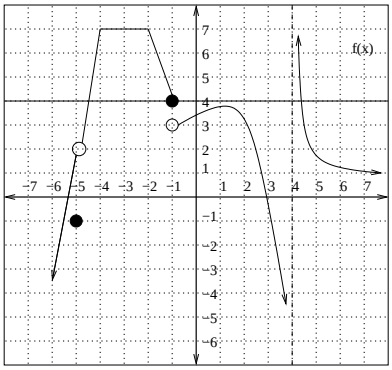


The graphs below represents a discontinuity at $x = 4$ because $\lim_{x\to 4} f(x) \neq f(4)$





EXAMPLE 1: For the graph of $f(x)$ given below, locate all discontiniuties. For each discontinuity, find the limit from the left and the limit from the right.



EXAMPLE 2: Explain why the following functions are or are not continuous at the indicated value of x. Support your answer.

(i) $f(x) = \dfrac{-1}{(1-x)^2}$, $x = 1$.

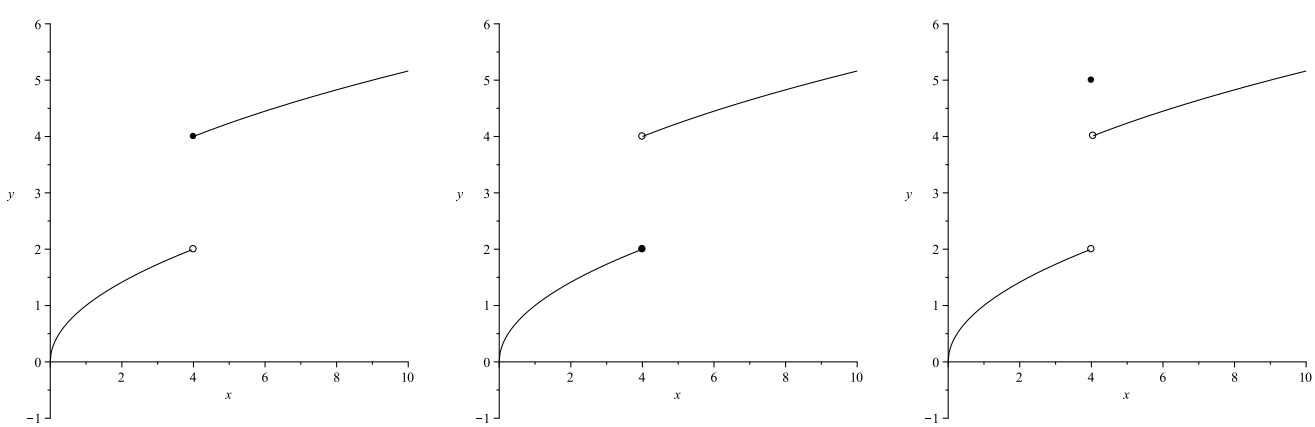
(ii) $f(x) = \begin{cases} \dfrac{x^2-2x-8}{x-4} & \text{if } x \neq 4 \\ 3 & \text{if } x = 4 \end{cases}$ at $x = 4$

(iii) $f(x) = \begin{cases} 2x+1 & \text{if } x \leq -1 \\ 3x & \text{if } -1 < x < 1 \\ x^2+2 & \text{if } x > 1 \\ 4 & \text{if } x = 1 \end{cases}$ at $x = \pm 1$

EXAMPLE 3: If $g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } \geq 4 \end{cases}$ For what value(s) of c is $g(x)$ continuous? In order to receive full credit, your answer must be fully supported by the definition of continuity.

EXAMPLE 4: If $f(x) = \begin{cases} x^2 + c & \text{if } x > 1 \\ 4 & \text{if } x = 1 \\ 4cx + 3 & \text{if } x < 1 \end{cases}$,For what value(s) of c is $f(x)$ continuous, if any? In order to receive full credit, your answer must be fully supported by the definition of continuity.

<u>Definition</u> We say $f(x)$ is continuous from the right at $x = a$ if $\lim_{x \to a^+} f(x) = f(a)$. Similarly, we say $f(x)$ is continuous from the left at $x = a$ if $\lim_{x \to a^-} f(x) = f(a)$.



<u>Intermediate Value Theorem</u> If $f(x)$ is continous on the interval $[a, b]$ and N is any number strictly between $f(a)$ and $f(b)$, then there is a number c, $a < c < b$, so that $f(c) = N$.

EXAMPLE 5: If $g(x) = x^5 - 2x^3 + x^2 + 2$, show there a number c so that $g(c) = -1$.

EXAMPLE 6: Show there is a root to the equation $x^5 - 2x^4 - x - 3$ on the interval $(2, 3)$.