Section 4.5: Exponential Growth and Decay

**Definition:** If $y(t)$ is the value of a quantity at time $t$ and if the rate of change of $y$ with respect to $t$ is proportional to its size $y(t)$ at any time, [That is $\frac{dy}{dt} = ky$], then the quantity $y(t)$ at time $t$ is given by

$$y(t) = y_0 e^{kt}$$

where $y_0$ is the initial quantity and $k$ is a constant. Given information, your primary goal is to find $k$.

**EXAMPLE 1:** A bacteria culture starts with 4000 bacteria and the population triples every half-hour.

(i) Find an expression for the number of bacteria after $t$ hours.

(ii) Find the number of bacteria after 20 minutes.

(iii) Find the rate of growth after 20 minutes.
**EXAMPLE 2:** Def: The **half-life** of a substance is the amount of time it takes for half of the substance to disintegrate. Polonium-210 has a half-life of 140 days.

(i) If a sample has a mass of 200 mg, find a formula for the mass that remains after \( t \) days.

(ii) When will the mass be reduced to 10 mg?

**EXAMPLE 3:** After 3 days a sample of radon-222 decayed to 58% of its original amount. What is the half-life of radon-222? How long will it take the sample to decay to 10% of its original amount?
EXAMPLE 4: A curve passes through the point $(0, 7)$ and has the property that the slope of the curve at every point $p$ is half the $y$-coordinate of $p$. Find the equation of the curve.

EXAMPLE 5: The rate of change of atmospheric pressure $P$ with respect to altitude $h$ is proportional to $P$, provided that the temperature is constant. At a specific temperature the pressure is 101 kPa at sea level and 86.9 kPa at $h = 1,000$ m. What is the pressure at an altitude of 3500 m?
Compound Interest: If \( A_0 \) dollars is invested at \( r\% \) compounded \( n \) times a year, then the amount in the account after \( t \) years is given by \( A = A_0(1 + r/n)^{nt} \).

EXAMPLE 6: If $4000 is invested at 8\% compounded monthly, how much money is in the account at the end of 6 years?

Continuous Compound Interest: If \( P \) dollars is invested at \( r\% \) compounded continuously, then the amount in the account after \( t \) years is given by \( A = Pe^{rt} \).

EXAMPLE 7: How much money should be invested now at 6\% compounded continuously in order to have $30,000 18 years from now?
**Definition:** The rate of cooling of an object is proportional to the temperature difference between the object and the temperature of the object’s surroundings. If $y(t)$ is the temperature of the object at time $t$, then $\frac{dy}{dt} = k(y - T)$, where $y$ is the temperature of the object at time $t$ and $T$ is the room temperature (the temperature of the room in which the object is cooling). The solution of this equation, which gives the temperature of the object at time $t$, is $y(t) = (y_0 - T)e^{kt} + T$, where $y_0$ is the initial temperature of the object.

**EXAMPLE 8:** A thermometer is taken from a room where the temperature is 20°C to the outdoors, where the temperature is 5°C. After one minute, the temperature reads 12°C. Use Newton’s Law of Cooling to answer the following questions.

a.) What will the reading of the thermometer be after 2 minutes?

b.) When will the thermometer read 6°C?