Section 5.1: What does $f'$ say about $f$?

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- If $f' > 0$ on an interval $I$, then $f$ is increasing on $I$.
- If $f' < 0$ on an interval $I$, then $f$ is decreasing on $I$.
- If $f'$ goes from positive to negative at $x = a$, and $x = a$ is in the domain of $f$, then $f$ has a local maximum at $x = a$.
- If $f'$ goes from negative to positive at $x = a$, and $x = a$ is in the domain of $f$, then $f$ has a local minimum at $x = a$.

Illustration:

[Graph of the derivative of $f$]
EXAMPLE 1: Below is the graph of the derivative, $f'$, of some function $f$. Use it to answer the following questions:

(i) On what intervals is $f$ increasing?  

(ii) On what intervals is $f$ decreasing?  

(iii) At what $x$ values does $f$ have a local maximum or minimum?
**Definition** If the slopes of a curve become progressively larger as $x$ increases, then we say $f$ is **concave upward**. If the slopes of a curve become progressively smaller as $x$ increases, then we say $f$ is **concave downward**.

Illustration:

What does $f''$ say about $f$?

- If $f'' > 0$ on an interval $I$, then $f'$ is increasing, hence $f$ is concave up on $I$.

- If $f'' < 0$ on an interval $I$, then $f'$ is decreasing, hence $f$ is concave down on $I$.

- If $f$ changes concavity at $x = a$, and $x = a$ is in the domain of $f$, then $x = a$ is an inflection point of $f$. 
EXAMPLE 2: If $f'(4) = 0$ and $f''(4) = 5$, what can be said about $f$?

EXAMPLE 3: If $f'(x) = e^{-x^2}$ what can be said about $f$?

EXAMPLE 4: Sketch a graph of $f$ satisfying the following conditions:

(i) $f'(x) > 0$ on the interval $(-\infty, 1)$ and $f'(x) < 0$ on the interval $(1, \infty)$.

(ii) $f''(x) > 0$ on the interval $(-\infty, -2)$ and $(2, \infty)$.

(iii) $f''(x) < 0$ on the interval $(-2, 2)$.

(iv) $\lim_{x \to -\infty} f(x) = -2$ and $\lim_{x \to \infty} f(x) = 0$. 
