Section 5.2: Maximum and Minimum Values

**Definition**

(1) A function \( f \) has an **absolute maximum** at \( c \) if \( f(c) \geq f(x) \) for all \( x \) in the domain of \( f \). A function \( f \) has an **absolute minimum** at \( c \) if \( f(c) \leq f(x) \) for all \( x \) in the domain of \( f \). In this case, we call \( f(c) \) the **maximum value** or **minimum value**, respectively.

(2) A function \( f \) has a **local maximum** at \( c \) if \( f(c) \geq f(x) \) when \( x \) is near \( c \). A function \( f \) has a **local minimum** at \( c \) if \( f(c) \leq f(x) \) when \( x \) is near \( c \).

**EXAMPLE 1**: Find all absolute and local extrema by graphing the function:

(a) \( f(x) = 1 - x^2, \ -3 < x \leq 2 \).

(b) \( f(x) = \begin{cases} x^2 & \text{if } -1 \leq x < 0 \\ 2 - x^2 & \text{if } 0 \leq x \leq 1 \end{cases} \)
**Definition** We call $x = c$ a **critical number** of $f(x)$ if $x = c$ is in the domain of $f$ and either $f'(c) = 0$ or $f'(c)$ does not exist.

**EXAMPLE 2**: Find the domain and critical values for the following functions:

(a) $f(x) = 4x^3 - 9x^2 - 12x + 3$

(b) $f(x) = |x^2 - 1|$

(c) $f(x) = \sqrt[3]{x^2 - 3x}$

(d) $f(x) = xe^{2x}$

(e) $f(x) = x \ln x$
**Extreme Value Theorem** If $f(x)$ is a continuous function on a closed interval $[a, b]$, then $f$ will attain both an absolute maximum and an absolute minimum.  

Graphical illustration:

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**EXAMPLE 3:** Use the extreme value theorem to find the absolute extrema for $f(x) = 1 + 27x - x^3$ on the interval $[0, 4]$.

**EXAMPLE 4:** Use the extreme value theorem to find the absolute extrema for $f(x) = x - 2 \cos x$ on the interval $[0, \pi]$.  