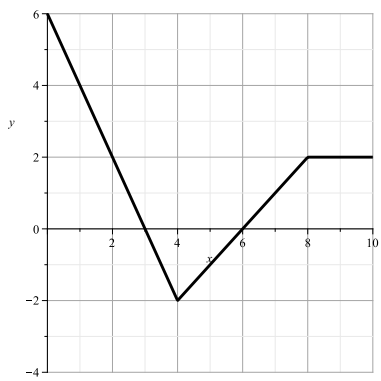
Section 6.4: The Fundamental Theorem of Calculus

<u>The Fundamental Theorem of Calculus Part I</u>: If f is continuous on $[a, b]$, then the function g defined by $g(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$.

EXAMPLE 1: If $g(x) = \int_0^x f(t)\,dt$, where the graph of $f(t)$ is given below, evaluate $g(3)$, $g(6)$ and $g(10)$.



EXAMPLE 2: Find $g'(x)$ for

(a) $g(x) = \int_{-1}^x \sqrt{t^3 + 1}\,dt$

(b) $g(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^2 + 1}\,dt$

(c) $g(x) = \int_{x^2}^3 e^{t^3}\,dt$

(d) $g(x) = \int_{x^2}^{\sin x} \frac{\cos t}{t}\,dt$

<u>The Fundamental Theorem of Calculus Part II</u>: If f is continuous on $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is an antiderivative of f.

EXAMPLE 3:

(a) Evaluate $\int_1^2 \frac{1}{x^2}\,dx$

(b) Evaluate $\int_{-1}^0 (5x^2 - 4x + 3)\,dx$

(c) Evaluate $\int_{\ln 3}^{\ln 6} 8e^t\,dt$

(d) Evaluate $\int_0^1 \frac{1}{x^2+1}\,dx$

(e) Evaluate $\displaystyle\int_1^2 \frac{x^6 - x^2}{x^7}\, dx$

(f) Evaluate $\displaystyle\int_0^2 (w^3 - 1)^2\, dw$

(g) Evaluate $\displaystyle\int_{-2}^0 |x^2 - 1|\, dx$

(h) Find $\int_0^{\pi/2} f(x)\,dx$ where $f(x) = \begin{cases} \cos x & \text{if } 0 \le x < \dfrac{\pi}{4} \\ \sin x & \text{if } \dfrac{\pi}{4} \le x \le \dfrac{\pi}{2} \end{cases}$

EXAMPLE 4: Suppose an object is moving according to velocity $v(t) = 3t - 5$, $0 \le t \le 3$. Find the displacement and distance traveled during the first 3 seconds.