Summer 2005

STEPS: Review of Pre-Calculus

courtesy: Amy Austin

Domain

1. Find the domain of the following functions. Enter your answer both as an inequality and in interval notation.

   a.) $f(x) = x^3 + 3x^2 + 1$

   b.) $f(x) = \dfrac{1}{x}$

   c.) $f(x) = \dfrac{x+1}{x^2 - 2x - 3}$

   d.) $f(x) = \sqrt{x}$

   e.) $f(x) = \sqrt[4]{2 - 3x}$

   f.) $f(x) = \sqrt{x^2 - 3x - 4}$

   g.) $f(x) = \sqrt{\dfrac{x}{x-2}}$

   h.) $f(x) = \dfrac{\sqrt{x+2}}{x^2 - x - 20}$

   i.) $f(x) = \dfrac{\sqrt[3]{x+2}}{x^2 + 1}$

Computations

2. For $f(x) = x^3 - 2x + 1$, find $f(3)$, $f(a+1)$ and $f(x+h)$.

3. For $f(x) = x^2 + x + 56$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

4. For $f(x) = 3 - 4\sqrt{x+1}$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

5. If $f(x) = \sqrt{x-3}$ and $g(x) = x^2 + 2$, find $f \circ g$, $g \circ f$ and their domains.

Linear and Quadratic Functions

6. Find the equation of the line passing through the points $(2, 3)$ and $(-5, 7)$. Express your answer in the form $y = mx + b$.

7. As dry air moves upward, it expands and cools. If the ground temperature is 26° C and the temperature at a height of 2 km is 15° C, express the temperature T as a function of the height h, assuming a linear model is appropriate. What is the temperature at a height of 3.5 km?

8. Graph the quadratic $f(x) = x^2 - 4x - 12$. Identify the vertex and intercept(s).

9. Graph the quadratic $f(x) = -x^2 + 4x - 5$. Identify the vertex and intercept(s).

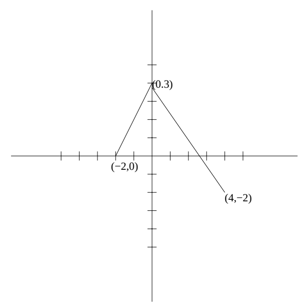
10. Sketch the region bounded by the curves $y = 9 - x^2$ and $y = 2x + 1$.

Piece-wise defined functions

11. Sketch the graph of

$$f(x) = \begin{cases} 2x + 1 & \text{if } x < -1 \\ 7 & \text{if } x = -1 \\ 9 - x^2 & \text{if } x > -1 \end{cases}$$

12. Write the following as a piece-wise defined function:



13. Write the following absolute value functions as piece-wise defined functions. Sketch the graph.

    a.) $f(x) = |x|$

    b.) $f(x) = |2x - 5|$

    c.) $f(x) = |x^2 - 2x - 3|$

    d.) $f(x) = x + 4|x| + 1$

Solving Inequalities

14. Solve $|x + 1| < 8$

15. Solve $|2x - 1| > 4$

16. Solve $\left|\dfrac{x+1}{3x-4}\right| = 2$

17. Solve $\dfrac{x+4}{x+7} \leq 2$

18. Solve $x^2 - 2x \leq 3$

Factoring

19. Factor the following expressions completely.

    a.) $x^2 - 64$

    b.) $3x - x^3$

    c.) $1 - 4x^4$

    d.) $x^2 + 2x - 3$

    e.) $x^3 - 1$

    f.) $x^3 + 216$

    g.) $x^4 - \frac{81}{16}$

Simplifying Rational Expressions

20. Simplify the following rational expressions completely.

    a.) $\frac{25 - x^2}{x - 5}$

    b.) $\frac{4}{x} - \frac{2}{x^2}$

    c.) $\frac{1}{x+3} + \frac{3}{x+2}$

    d.) $\frac{\frac{1}{x} + 1}{\frac{1}{x} - 1}$

    e.) $\frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h}$

Solving equations

21. Solve for x:

    a.) $4x - 11 = 3x + 1$

    b.) $(2x - 1)(4x - 3) = (8x - 1)(x + 2)$

    c.) $(x - 2)^4 = 16$

    d.) $(x + 1)^3 = -8$

    e.) $\frac{x+3}{5} = \frac{2-x}{7}$

    f.) $\frac{2x+5}{x+1} = \frac{3}{4}$

    g.) $1 - \frac{3}{x+1} = -5$

    h.) $\frac{3x}{x-2} = 1 + \frac{6}{x-2}$

Trigonometry

22. Convert $34°$ to radians.

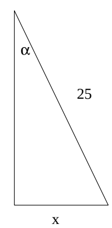
23. Convert $\frac{\pi}{15}$ radians to degrees.

24. If the radius of a circle is 5 inches, what is the length of an arc subtended by a central angle of $60°$?

25. If $\sin(\theta) = \frac{4}{5}$, and $0 \leq \theta \leq \frac{\pi}{2}$ find $\cos(\theta)$, $\tan(\theta)$,$\sec(\theta)$,$\csc(\theta)$, and $\cot(\theta)$.

26. If $\csc(\theta) = -\frac{4}{3}$, and $\frac{3\pi}{2} \leq \theta \leq 2\pi$, find $\sin(\theta)$, $\cos(\theta)$,$\sec(\theta)$,$\tan(\theta)$, and $\cot(\theta)$.

27. Refer to the figure to find the value of x, given that $\alpha = 20°$.



28. It is imperative that you know the value of all six trig functions evaluated at $0°$, $30°$,$45°$,$60°$, and $90°$ BY HEART! If you know these, then you can easily count around the unit circle to obtain, say, $\sin(150°)$ by using a reference angle. The reference angle is by definition the smallest angle made with the x axis. Now I will do streaming video of how this is done.

29. Solve the following equations for x:

    a.) $2\cos(x) = 1$ in the interval $[0, 2\pi]$

    b.) $2\cos^2 x = 1$ in the interval $[-\pi, \pi]$

    c.) $\sin(2x) = \cos x$ in the interval $[0, 3\pi]$.