

## 6.1 Sets and Set Operations

- A **set** is a collection of objects. The objects in a set are called the **elements** of the set. Usually sets are denoted by uppercase letters,  $A, B, C, \dots$ , and elements are usually denoted by lowercase letters,  $a, b, c, \dots$ .

There are two ways to write a set: roster notation and set-builder notation.

- **Roster notation** is where the set is written by listing all the elements of a set inside braces such as  $A = \{a, e, i, o, u\}$ .
- **Set-builder notation** is where the set is written in terms of a rule or property that describes all the elements in the set.

Write the set  $A$  above in set-builder notation:

- If  $a$  is an element of a set  $A$ , then we write  $a \in A$ . If  $a$  is not an element in  $A$ , then  $a \notin A$ .  
If  $A = \{1, 2, 3, 5, 7\}$ , then  $3 \in A$  but  $4 \notin A$ .
- Two sets  $A$  and  $B$  are **equal**,  $A = B$  if and only if they have exactly the same elements. It doesn't matter what order they are written in.  
If  $A = \{1, 3, 4, 7, 9\}$  and  $B = \{3, 7, 4, 9, 1\}$ , then  $A = B$ .
- We say a set  $A$  is a **subset** of  $B$  if every element of  $A$  is also an element of  $B$ . We denote this by  $A \subseteq B$ .  
If  $A = \{1, 3, 4, 7, 9\}$ ,  $B = \{3, 7, 4, 9, 1\}$ , and  $C = \{1, 4, 7\}$ , list all subset relationships.
- If  $A \subseteq B$  but  $A \neq B$ , then we say that  $A$  is a **proper subset** of  $B$ , and we denote this by  $A \subset B$ . In other words,  $A$  is essentially a 'smaller' subset of  $B$ . List all proper subset relationships from above.

Make sure you know the difference between  $\in$  and  $\subseteq$ .  $\in$  is used when we are talking about an ELEMENT being in a set.  $\subseteq$  is used when we are talking about a SET being a subset of another set. Which of the following is a TRUE statement if  $A = \{1, 2, 3, 4\}$  and  $B = \{2, 4\}$ .

$2 \in A$      $2 \subset A$      $\{2\} \subseteq A$      $\{4\} \subset B$      $\{3\} \in A$      $B \subset A$      $B \subseteq A$      $B \subset B$      $A \subseteq A$

- The **empty set** is the set that contains no elements and is denoted by  $\emptyset$  or by  $\{\}$ .  
FACT: The empty set is a subset of EVERY set. For any set  $A$ ,  $\emptyset \subseteq A$  or  $\{\} \subseteq A$ .  
Example: List ALL subsets of the set  $C = \{1, 4, 7\}$ .

If a set  $A$  has  $n$  elements in it, then the total number of subsets of  $A$  is  $2^n$ .

The number of proper subsets of  $A$  is \_\_\_\_\_.

If  $A$  contains 4 elements, how many subsets does  $A$  have? \_\_\_\_\_ Proper subsets? \_\_\_\_\_

- A **universal set** is the set of all elements of interest in a particular discussion. It can vary depending on the problem.
- A **Venn diagram** is a visual representation of a set.

Examples: Draw Venn diagrams to illustrate the following scenarios.

$A$  is a proper subset of  $B$ . ( $A \subset B$ ).

$A$  and  $B$  have no elements in common.

## Set Operations

- The **union** of two sets  $A$  and  $B$ , written  $A \cup B$ , is the set of all elements that are IN  $A$  OR  $B$  OR BOTH. This is the analog to  $\vee$ , the inclusive disjunction, in logic.

- The **intersection** of two sets  $A$  and  $B$ , written  $A \cap B$ , is the set of all elements that  $A$  and  $B$  have in common. In other words, it is the set of elements that are IN BOTH  $A$  AND  $B$  at the same time. This is the analog to  $\wedge$ , the conjunction, in logic.

If two sets  $A$  and  $B$  have no elements in common, then  $A \cap B = \emptyset$  and we say  $A$  and  $B$  are **disjoint**.

- The **complement** of a set  $A$ , written  $A^c$  is the set of all elements that are NOT IN  $A$  (but still in the universal set  $U$  of the problem). This is the analog to  $\sim$ , the negation, in logic.

Example: Let  $U = \{n, 2, 3, 4, w, 6, 7, 8, 9\}$ ,  $A = \{n, w, 7\}$ ,  $B = \{x \mid x \text{ is an even number between 1 and 9}\}$ ,  $C = \{n, 3, 4, 9\}$ . Find the following sets.

- $A \cup B$
- $A \cap C$
- $C^c$
- $A \cap (B \cup C)^c$
- $(A^c \cap C) \cup B^c$

Sets:  $U = \{n, 2, 3, 4, w, 6, 7, 8, 9\}$ ,  $A = \{n, w, 7\}$ ,  $B = \{x | x \text{ is an even number between 1 and 9}\}$ ,  
 $C = \{n, 3, 4, 9\}$

- $(A \cup B \cup C)^c$

- $(A \cap B \cap C)^c$

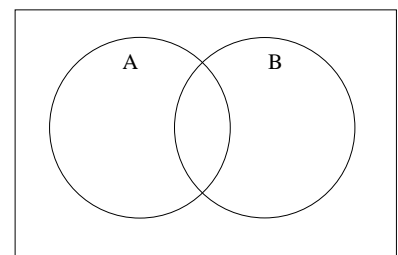
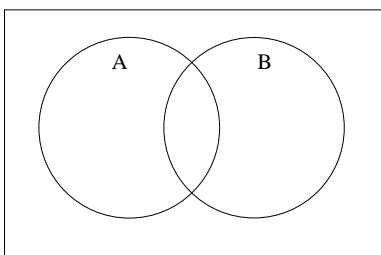
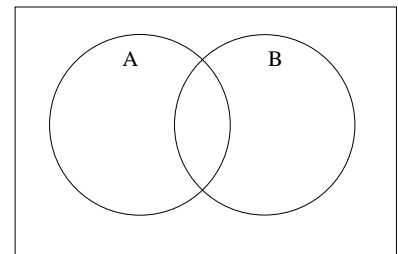
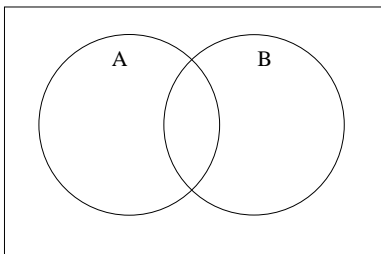
**Properties of Set Operations**

1.  $U^c = \emptyset$  and  $\emptyset^c = U$
2.  $(A^c)^c = A$
3.  $A \cup A^c = U$
4.  $A \cap A^c = \emptyset$

De Morgan's Laws:

- $(A \cup B)^c = A^c \cap B^c$
- $(A \cap B)^c = A^c \cup B^c$

You can think of these De Morgan's Laws as a kind of distributive property for sets.  
 Verify the first De Morgan Law using a Venn diagram.



Consider the following sets. Let  $U$  be the universal set of all undergraduate students at Texas A&M.

$$M = \{x \in U | x \text{ is a male}\}$$

$$F = \{x \in U | x \text{ is a freshman}\}$$

$$S = \{x \in U | x \text{ is a senior}\}$$

Using these three sets, write the set that represents the following statements.

The set of students at A&M who are male freshman.

The set of students at A&M who are male seniors or female freshmen.

The set of female students at A&M who are not seniors.

The set of students at A&M who are neither freshman nor seniors.

What do the following sets represent in words?

$$M \cup F^c$$

$$M \cap (F \cup S)$$

Consider the following 3 sets in the same universal set of undergraduate students at A&M.

$$I = \{x \in U | x \text{ has an iPod}\}$$

$$D = \{x \in U | x \text{ has a digital camera}\}$$

$$L = \{x \in U | x \text{ has a laptop}\}$$

Using these three sets, write the set that represents the following statements.

The set of students at A&M who have an iPod and a digital camera but not a laptop.

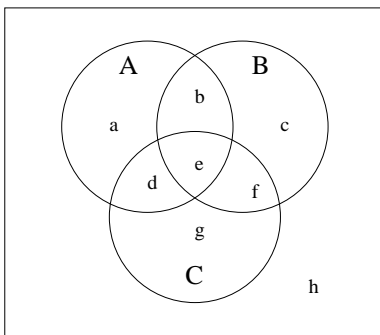
The set of students at A&M who only have a digital camera.

What does the set  $I \cup D \cup L$  represent in words?

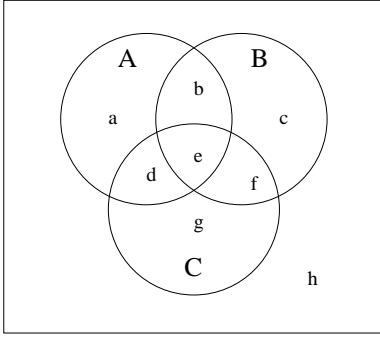
### Shading Venn Diagrams

Shade the appropriate region in a 3-circle Venn diagram.

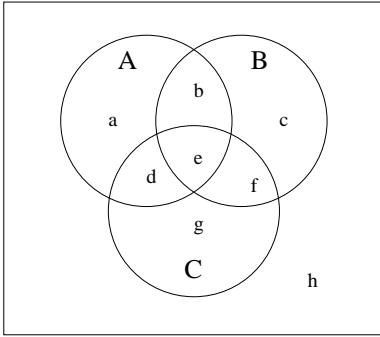
$$A \cap B^c \cap C^c$$



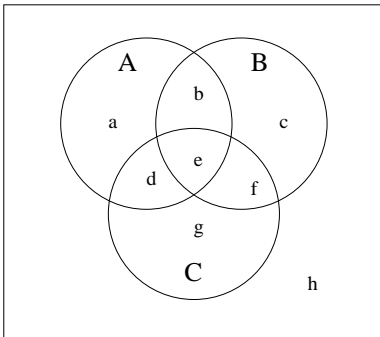
$$(A \cap C) \cup (B \cap C^c)$$



$$(B^c \cup C) \cap (A \cup C^c)$$



$$(A^c \cup B)^c \cap C$$



## 6.2 The Number of Elements in a Set

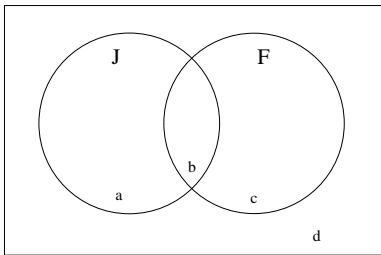
We denote the number of elements in a set  $A$  as  $n(A)$ .

If  $A = \{1, 6, a\}$ , then  $n(A) = \underline{\hspace{2cm}}$ .

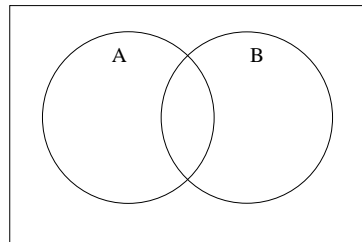
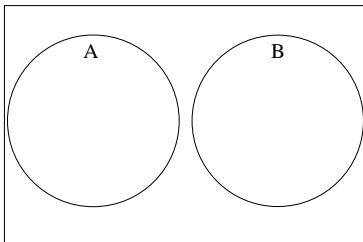
If  $B = \{x \mid x \text{ is a consonant in the alphabet } \}$ , then  $n(B) = \underline{\hspace{2cm}}$

$n(\emptyset) = \underline{\hspace{2cm}}$

Example: In a room of 100 students, 30 are juniors, 50 are female, and 10 are female juniors. Fill in the Venn diagram below with the appropriate number of students in each region. Describe the students in region  $a$ .



Counting  $n(A \cup B)$ :



### Union Rule

If  $A$  and  $B$  are any two finite sets, then the following formula holds.

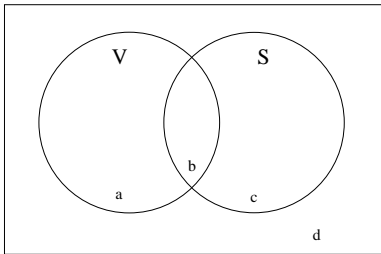
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

If  $A$  and  $B$  are disjoint, the formula reduces to:

$$n(A \cup B) = n(A) + n(B)$$

Example: People were asked what kind of condiments they like on their hamburger. 60 people said they like ketchup and 70 people said they like mustard. If 100 people like ketchup or mustard, how many people like both?

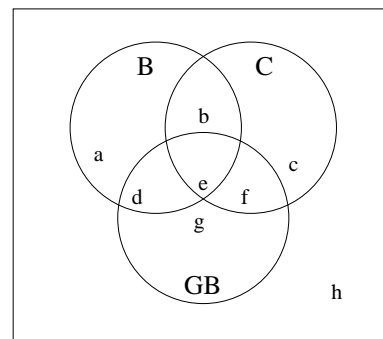
Example: 50 people were surveyed about whether they like the ice cream flavors vanilla and strawberry. 20 people liked vanilla. 25 people liked strawberry. 30 people liked vanilla or strawberry.



- (1) How many people liked vanilla and strawberry?
- (2) How many people like exactly one of these two flavors?
- (3) How many people liked strawberry or did not like vanilla?

Example: A survey was done of 200 8-year olds asking if they like certain types of vegetables (broccoli, carrots, green beans). The following results were obtained:

- 28 kids only liked green beans.
- 73 kids liked broccoli.
- 40 kids liked broccoli and green beans.
- 79 kids liked exactly 2 of these vegetables.
- 12 kids liked broccoli and carrots, but not green beans.
- 34 kids only liked carrots.
- 104 kids liked carrots or green beans, but not broccoli.



- (1) How many kids liked carrots, but not green beans?
- (2) How many kids liked exactly 1 of the three vegetables?
- (3) How many kids do not like carrots?



Example: A survey was conducted of students at a liberal arts college to determine the foreign language courses they had taken while undergraduates at the college. The results are given below.

200 had taken a Spanish class.

$$n(F \cap G) = 18$$

316 had taken a Spanish or a German class.

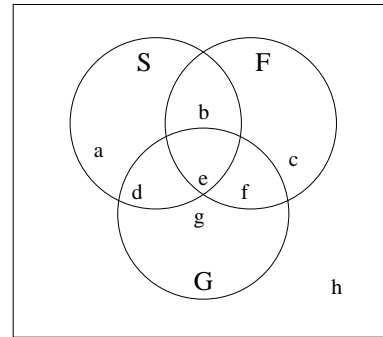
282 had taken a French or a German class, but not both.

69 had taken a class in at least 2 of these 3 languages.

135 had not taken a Spanish or a French class.

$$n(S \cap F \cap G^c) = 30$$

3 had taken classes in all 3 languages.



(1) How many had taken a class in at least one of the three languages?

(2) Calculate  $n((S \cap G) \cup F^c)$ .