

Abstracts of invited talks

Nodal surfaces and injectivity sets

Mark Agranovsky
Bar Ilan University

Nodal sets are zero loci of Laplace eigenfunctions. The geometry of a single nodal set may be very complicated. On the other hand, simultaneous vanishing of many eigenfunctions is an overdetermined condition and hence the common nodal set of a large family of eigenfunctions either is small or, if not, should have a special geometric shape. The talk is devoted to the problem of describing common nodal hypersurfaces for families of eigenfunctions in Euclidean space and to the equivalent problem of describing injectivity sets for the spherical mean transform.

Periodic approximant to 1D-aperiodic Hamiltonians

Jean V. Bellissard
Georgia Institute of Technology

So far the most efficient way to compute numerically the spectrum of a Schrödinger operator with an aperiodic potential is to use periodic approximations. The main question investigated here will be to understand in what sense these approximations are valid and if it is possible to use this method to draw mathematical conclusions concerning the limiting operators. Various motivations for this problem will be presented, in connection with the spectral properties of quasicrystals. Then a review of the methods used so far to analyse rigorously the spectral properties of one-dimensional Schrödinger's operators with aperiodic potentials. Several models will also be given to illustrate the problem. The Anderson-Putnam complex, in the version provided by Franz Gähler, called here GAP-graphs, will be described more precisely. Several properties will be described, such a complexity, and loops. Using the technique of continuous fields of operators, motivated by the concept of continuous fields of C^* -algebras, several convergence results will be provided. A discussion of the methods of proofs and of open problems will conclude the talk.

Transport based imaging in random waveguides

Liliana Borcea
University of Michigan

I will describe a study of imaging in random waveguides in strong scattering regimes where the waves are nearly incoherent. We can obtain meaningful results if we work with cross-correlations of such waves. I will explain how we can calculate these cross-correlations and show how to relate them to the unknown in inverse problems via a discrete system of radiative transport equations. I will show how we can invert these equations and quantify explicitly the negative effects of scattering in inversion.

Lagrangian variational framework for boundary value problems

Alexander Figotin and Guillermo Reyes

University of California, Irvine

A boundary value problem is commonly associated with constraints imposed on a system at its boundary. We advance here an alternative point of view treating the system as interacting boundary and interior subsystems. This view is implemented through a Lagrangian framework that allows to account for (i) a variety of forces including dissipative acting at the boundary; (ii) a multitude of features of interactions between the boundary and the interior fields when the boundary fields may differ from the boundary limit of the interior fields; (iii) detailed pictures of the energy distribution and its flow; (iv) linear and nonlinear effects. We provide a number of elucidating examples of the structured boundary and its interactions with the system interior. We also show that the proposed approach covers the well known boundary value problems.

Asymptotics of Steklov eigenvalues in large parameter

Leonid Friedlander

University of Arizona

Let Ω be a bounded planar domain with a smooth boundary. I develop asymptotics of Steklov eigenvalues of the operator $-\Delta + \lambda$ in Ω as $\lambda \rightarrow \infty$.

Isospectrality of Dirichlet-to-Neumann operators

Carolyn Gordon

Dartmouth College

Abstract: The Dirichlet-to-Neumann operator of a compact Riemannian manifold M with boundary is a linear map $C^\infty(\partial M) \rightarrow C^\infty(\partial M)$ that maps the Dirichlet boundary values of each harmonic function f on M to the Neumann boundary values of f . The spectrum of this operator is discrete and is called the Steklov spectrum. We will discuss joint work with Peter Herbrich and David Webb concerning the construction of pairs of Steklov isospectral bounded domains in a fixed noncompact manifold. The Laplacians on these domains are also isospectral for both the Dirichlet and Neumann boundary problems and the exterior domains are isophasal. The latter result is joint with Peter Perry.

Radon transform and spherical functions

Sigurdur Helgason

Massachusetts Institute of Technology

The injectivity of the Radon transform on Euclidean space comes from the Fourier Transform. On a symmetric space G/K the analogous question is settled by means of the spherical function, in particular its boundedness. I will discuss the analogous question for the Cartan motion group. Here the problem is unsolved.

Discrete geometric analysis applied to structural understanding of materials

Motoko Kotani

Tohoku University

Discrete Geometric Analysis concerns relations between discrete and continuum. I would like to explain the mathematical tools to apply to materials research and some emerging results. Main issue is how one can understand the relation between local structure and global materials properties. There are many interesting problems there.

Stability and finite propagation for biomechanical imaging - using linear solid (viscoelastic) models

Joyce McLaughlin

Rensselaer Polytechnic Institute

Biomechanical imaging in tissue is based on the coupled physics experiment where tissue displacement (on the order of tens of microns) is induced by either: (1) by single frequency oscillation; or (2) by a pulse that induces a propagating wave with a front. During the displacement, a sequence of B-scan data sets are acquired or a sequence of MR spectral data sets are acquired. Data processing of those data sets yields movie(s) of one or more displacement components. Utilizing these movie(s), and assuming the tissue is viscoelastic, the goal is to image shear properties of the tissue. Two important questions arise. One is: Is there a stability result based on one set of movies for two or three components? And a second question is: How general can one make the plane strain or fully 3D Linear Solid Viscoelastic Model (for example, what anisotropy characteristics can be included) and still be modeling a medium that has finite propagation speed. This talk will address both of these issues.

A characterization of the response of electromagnetic circuits

Graeme Milton
University of Utah

Electromagnetic circuits are the electromagnetic analog of an elastic spring-mass network, but operate only in a narrow frequency band. They consist of thin triangular magnetic components joined at the edges by cylindrical dielectric components. A complete characterization of their possible responses is obtained both in the lossless and lossy cases. We also give a complete characterization, as a function of frequency, of more normal circuits, electromagnetic loop circuits, consisting of connected loops of material with high dielectric constant, linked by loops of material with high magnetic permeability. This is joint work with Pierre Seppecher.

Solving two-dimensional integrable dispersive equations by the inverse scattering method

Peter Perry
University of Kentucky

The inverse scattering method has been applied with remarkable success to solve completely integrable partial differential equations such as the celebrated Korteweg-de Vries equation, and the cubic nonlinear Schrödinger equation, in one space and one time dimension. For these equations, inverse scattering provides a kind of “nonlinear Fourier transform” that yields explicit solutions and allows one to compute the asymptotic behavior of solutions in space and time. At the core of inverse scattering problem lies a Riemann-Hilbert problem depending on the space and time variables as parameters.

In this talk, we’ll discuss recent progress in applying the inverse scattering method to completely integrable equations in two space and one time dimensions. We will focus on the Davey-Stewartson equation, a completely integrable analogue of the cubic NLS in one-dimension, which describes the evolution of surface waves. At the core of the method is a $\bar{\partial}$ problem, the counterpart in this setting of the Riemann-Hilbert problem.

Inverse problem of optical tomography

John Schotland
University of Michigan

The inverse problem of optical tomography consists of recovering the spatially-varying absorption of a highly-scattering medium from boundary measurements. In this talk we will discuss direct reconstruction methods for this problem that are based on inversion of the Born series. Analogous results for the Calderon problem of reconstructing the conductivity in electrical impedance tomography will also be presented.

Spectral statistics of random Bernoulli matrix ensembles – a random walk approach.

Uzy Smilansky
Weizmann Institute of Science

We define a random walk on a graph whose vertices are in one-to-one to the matrices belonging to a given Bernoulli ensemble. Vertices (matrices) are adjacent if their difference is of minimal rank. We show that the induced spectral random walk can be described in terms of a Fokker-Planck equation, whose stationary solution provides the spectral joint probability distribution function.

Travel time tomography with partial data

Gunther Uhlmann
University of Washington

We will consider the inverse problem of determining the sound speed or index of refraction of a medium by measuring the travel times of waves going through the medium. This problem arise in several applications in geophysics and medical imaging among others.

The problem can be recast as a geometric problem: Can one determine a Riemannian metric of a Riemannian metric with boundary by measuring the distance function between boundary points? This is the boundary rigidity problem. We will also consider the problem of determining the metric from the scattering relation, the so-called lens rigidity problem. The linearization of these problems involve the integration of a tensor along geodesics, similar to the X-ray transform.

We will also describe some recent results, join with Plamen Stefanov and Andras Vasy, on the partial data case, where you are making measurements on a subset of the boundary.

Weyl law for signed counting function of positive interior transmission eigenvalues

Boris Vainberg
University of North Carolina, Charlotte

We study the relation between the scattering problem and the corresponding eigenvalue problem in the bounded region occupied by the obstacle. The latter problem in the case of scattering by a soft or rigid obstacle is the eigenvalue problem for the Dirichlet (or Neumann, respectively) Laplacian. We consider the interior transmission eigenvalue (ITE) problem, which arises when scattering by inhomogeneous media is studied. The ITE problem is not self-adjoint. However, we will show that it has a large set of positive eigenvalues. Moreover, we will obtain a signed version of the Weyl formula for the counting function of positive ITEs, where the eigenvalues are counted with plus or minus signs. The signs are observable and defined by the direction of motion of the related eigenvalues of the scattering matrix (when the latter approach $z=1$). The same technique is applicable to study exceptional sets in Faddeev's scattering problem. These are joint results with E. Lakshtanov.

Counting nodal domains on negatively curved surfaces.

Steven Zelditch

Northwestern University

We show that for non-positively curved surfaces with concave boundary (Sinai billiards) that the number of nodal domains of Dirichlet or Neumann eigenfunctions tends to infinity with the eigenvalue (except possibly for a sparse subsequence of exceptions). We prove that same result for negatively curved surfaces without boundary with with anti-holomorphic involutions. This is joint work with Junehyuk Jung.