

Concepts to Know #1

Math 141

1.3-1.5, 2.1-2.6

- **The Cartesian Coordinate System**

x and y axis (label your scales on graphs)

Ordered pairs (points): (x, y)

Origin: $(0,0)$

- **Straight Lines**

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Positive slope: line rises from lft to rt

Negative slope: line falls from lft to rt

Zero slope: Horizontal line

Undefined Slope: Vertical line

Equations of Lines

Pt-Slope Form: $y - y_1 = m(x - x_1)$

Slope-Int Form: $y = mx + b$

General Form: $Ax + By + C = 0$

Horizontal line: $y = a$

(a is the y -value of every point on the line)

Vertical line: $x = b$

(b is the x -value of every point on the line)

Intercepts

x -intercept = point where line crosses x -axis:

$$(\#,0)$$

y -intercept = point where line crosses y -axis:

$$(0,\#)$$

Parallel Lines $\implies m_1 = m_2$

(same slopes/diff. y -int)

- **1.3 - Linear Functions and Math. Models**

Functions

Domain

Range

Independent Variable

Dependent Variable

Linear Functions

Linear Depreciation: $V(t) = mt + b$

$|m|$ = rate of depreciation

b = value of asset at time zero

Scrap Value = lowest value asset attains

Linear Cost, Revenue and Profit

Cost: $C(x) = cx + F$ where c is the cost to produce each unit and F is the fixed costs

Revenue: $R(x) = sx$ where s is the selling price of each unit

Profit: $P(x) = R(x) - C(x)$

Linear Supply and Demand

$$S(x) = p = mx + b$$

$$D(x) = p = mx + b$$

**All points on supply and demand curves are of the form

$$(x, p) = (\text{quantity, price})!!**$$

- **1.4 - Intersection of Straight Lines**

Break-Even Point

$$R(x) = C(x) \text{ (or } P(x) = 0 \text{ to find quantity)}$$

x = break-even quantity

y = break-even revenue

Equilibrium Point

Supply = Demand

x = equilibrium quantity

y = equilibrium price

- **1.5 - The Method of Least Squares/Linear Regression**

Be able to use LinReg on your calculator to find the least-sq. line

Correlation coefficient (r) - determines the amount of the data explained by the line

(Want $|r|$ close to 1)

Be able to predict values, using the least-sq. line

• 2.1 - Systems of Linear Equations

Two linear eqns \implies three cases

Unique Soln (intersecting lines $\implies m_1 \neq m_2$)

No Soln (parallel lines \implies

$$m_1 = m_2 \text{ and } b_1 \neq b_2)$$

Infinitely Many Solns (same line \implies

$$m_1 = m_2 \text{ and } b_1 = b_2)$$

General soln (parametric soln)

Specific solns

Setting up systems of equations

DEFINE YOUR VARIABLES!

• 2.2/2.3 - Solving System of Equations

Gauss-Jordan (GJ) Elimination (Row Operations)

Interchange any two equations

Multiply an eqn by a non-zero constant

Add a multiple of one eqn to another

Reduced Row-Echelon Form

All zero rows must be below all non-zero rows

The first non-zero entry in each row is 1
(leading 1)

In any two successive (non-zero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row

If a column contains a leading 1, then the other entries in that column are zeros

Solving a System

Put system in “nice” form (align variables/constants)

Place “nice” system in an augmented matrix

Use RREF on your calculator to reduce your system

Read system (eqns) from reduced system

Find soln

• 2.4 - Matrices

Size (dimension): $m \times n$ ($m = \#$ rows, $n = \#$ columns)

Matrix elements: a_{ij} (element in row i and col j)

Equality \implies all corresponding entries equal

Addition/Subtraction (matrices must be the same size)

Matrix Transpose (A^T): switch rows and cols

Scalar Multiplication: multiply every entry by the scalar

• 2.5 - Multiplication of Matrices

The $\#$ of cols in the left matrix must equal the $\#$ of rows in the right matrix.
(Inner dimensions equal.)

If A is $(m \times n)$ and B is $(n \times p)$, then AB is $(m \times p)$.
(Outer dimensions give answer size.)

ORDER IS IMPORTANT!

Know how to multiply matrices by hand.

Identity Matrix: Square matrix ($\#$ rows = $\#$ cols) with 1's along the diagonal (from upper left to lower right) and 0's elsewhere

Be able to represent a system of eqns as a matrix eqn: $AX = B$

Put system in “nice” form

A = coefficient matrix

X = variable matrix

B = constant matrix

• 2.6 - Inverse of a Square Matrix

Matrix must be square

Not all matrices have an inverse
(“singular” means no inverse)

Inverse of $A = A^{-1}$

$$AA^{-1} = A^{-1}A = I$$

A system in form $AX = B \implies X = A^{-1}B$ if the inverse exists

Know how to solve a matrix equation